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A Winkler model approach for vertically and laterally loaded piles in nonhomogeneous soil

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SUMMARY

An investigation is made to present analytical solutions provided by a Winkler model approach for the analysis of single piles and pile groups subjected to vertical and lateral loads in nonhomogeneous soils. The load transfer parameter of a single pile in nonhomogeneous soils is derived from the displacement influence factor obtained from Mindlin's solution for an elastic continuum analysis, without using the conventional form of the load transfer parameter adopting the maximum radius of the influence of the pile proposed by Randolph and Wroth. The modulus of the subgrade reaction along the pile in nonhomogeneous soils is expressed by using the displacement influence factor related to Mindlin's equation for an elastic continuum analysis to combine the elastic continuum approach with the subgrade reaction approach. The relationship between settlement and vertical load for a single pile in nonhomogeneous soils is obtained by using the recurrence equation for each layer. Using the modulus of the subgrade reaction represented by the displacement influence factor related to Mindlin's solution for the lateral load, the relationship between horizontal displacement, rotation, moment, and shear force for a single pile subjected to lateral loads in nonhomogeneous soils is available in the form of the recurrence equation. The comparison of the results calculated by the present method for single piles and pile groups in nonhomogeneous soils has shown good agreement with those obtained from the more rigorous finite element and boundary element methods. It is found that the present procedure gives a good prediction on the behavior of piles in nonhomogeneous soils.

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KEY WORDS: Winkler model; piles and pile groups; non-homogeneity; vertical and lateral loads; soil-pile interaction; stiffness coefficient

1. INTRODUCTION

For the design methods of pile foundations, special attention has been recently concentrated not only on the bearing capacity but also on the settlement and horizontal displacements of a foundation under various loading conditions. The parametric solutions of piles have been produced for a variety of practical cases, and most of them are concerned with homogeneous soils. For a typical example, the integral equation method or the boundary element method (BEM) given by Butterfield and Banerjee [1] has been used to provide the numerical computer-based solutions for piles in homogeneous soils. For nonhomogeneous soils, Banerjee and Davies [2, 3], Banerjee [4], Poulos [5, 6], Poulos and Davis [7], Chow [8–10], Lee [11], Ta and Small [12–14], Zhang and Small [15], Small and Zhang [16], and Kitiyodom and Matsumoto [17] presented the solutions of pile foundations, using the numerical methods such as the finite element method (FEM) and BEM and the simplified analytical approaches.

The rigorous analytical approach ensures the boundary, the continuity, and the compatibility conditions of the displacement, stress, force, rotation, and moment for the pile and the half-space. The

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fundamental work for the rigorous analytical approach is due to Muki and Sternberg [18, 19], who investigated the diffusion and transfer of the axial load from a long cylindrical elastic bar into the surrounding elastic medium. Luk and Keer [20] presented a rigorous analytical formulation for the problem of a rigid cylindrical inclusion partially embedded in an isotropic elastic half-space in the case of the axial loading. Apirathvorakij and Karasudhi [21] investigated a rigorous analytical approach for the quasi-static bending of a pile embedded in a saturated porous half-space. Selvadurai and Rajapakse [22] demonstrated a rigorous analytical method related to the axial, lateral, and rotational loadings of a rigid cylindrical inclusion embedded in an isotropic elastic half-space. Rajapakse and Shah [23, 24] presented a rigorous analytical method to solve an elastic bar embedded in an elastic half-space subjected to longitudinal, lateral, and rotational loads.

However, the less rigorous analytical approach has several limitations, such as the absence of displacement, stress, force, rotation, and moment and the neglect of boundary, continuity, and compatibility conditions. A less rigorous approach was presented by Spillers and Stoll [25] to investigate the behavior of laterally loaded piles under static loading. They assumed that the pile was modeled as an elastic line that obeys the Bernoulli–Euler beam theory, and the interactive relationship between the beam and the half-space was modeled by Mindlin's solution. Improvements on the work of the less rigorous approach have been reported by many researchers. Poulos [5, 6, 26–28] and Poulos and Davis [7] used a finite difference method to analyze the behavior of vertically and laterally loaded single piles and pile groups. Randolph and Wroth [29] presented approximate closed-form linear elastic solutions for the settlement of a pile in homogeneous and nonhomogeneous soils. Randolph and Wroth [30] developed the method of using the closed-form solution for a vertically loaded single pile to produce the solution for vertically loaded pile groups. Poulos and Davis [7] presented both an analytical solution for a single floating pile and a solution based on an iterative method for a single end-bearing pile in nonhomogeneous soils by introducing an average elastic modulus for the soil.

Another less rigorous but sophisticated analytical approach was proposed by Tajimi [31]. This approach is that the pile is modeled as an elastic beam and the interactive relationship between the pile and the half-space adopts the functions expressed by the Fourier series instead of Mindlin's solution. Using the method proposed by Tajimi [31], Nogami and Novak [32, 33] presented a solution for an elastic pile embedded in a soil layer of equal height resting on a rigid base. Developing the methods by Tajimi [31], Nogami and Novak [32, 33], Nielsen [34] proposed a rigorous analytical approach based on the 3-D conditions and presented the characteristics of the resistance of an elastic layer to a horizontally vibrating pile. Baguelin *et al.* [35] investigated the lateral reaction mechanism of piles to provide the modulus of the subgrade reaction in an elastic medium. In the articles presented by Tajimi [31], Nogami and Novak [32, 33], Nielsen [34], and Baguelin *et al.* [35], the lateral pressure of a pile is expressed by the horizontal components of the radial normal stress and the shear stress around the pile. Subsequently, the lateral reaction acting on the circumference of a pile is obtained from the integral of the lateral pressure. Because the radial normal stress and the shear stress are related to the displacement, the lateral reaction may be expressed using the displacement. According to the method adopted by Tajimi [31], Nogami and Novak [32, 33], and Nielsen [34], although the lateral reaction can be expressed theoretically by the Fourier coefficients of the displacement in the Fourier series, the lateral reaction cannot be related to the displacement directly. Therefore, as a less rigorous approach, a Winkler model may be approximately assumed through the modulus of the subgrade reaction for the relationship between the lateral pressure and the displacement.

In developing the Winkler model of soil reaction for interaction between piles and soils in layered soils for vertically loaded piles, Mylonakis and Gazetas [36] proposed analytical expressions for settlement and interaction factors. Mylonakis [37] analytically investigated the modulus of the subgrade reaction on the basis of the Winkler model for axially loaded piles. The pile subjected to lateral loads is usually assumed to be governed by the simple theory of bending of beams. The analytical solutions of the governing equation for the deflection of a laterally loaded pile have been obtained by Chang [38] and Hetenyi [39]. The closed-form solutions, which Chang and Hetenyi proposed and are used in the subgrade reaction analysis, have been adopted to predict the load deflection for a laterally loaded pile in homogeneous soils. For piles in a two-layer system, solutions for the case where the modulus of the subgrade reaction varies linearly with depth have been given by Davisson and Gill [40] and Reddy and Valsangkar [41]. Using Mindlin's solution for lateral loads, Yamahara [42] proposed the lateral stiffness coefficients analytically at the mid-depth of a pile with circular section in a homogeneous soil. To analyze the behavior of piles subjected to lateral loads, Elahi *et al.* [43] and Liyanapathirana and Poulos [44] proposed Winkler-type methods where the modulus of the subgrade reaction of the Winkler model is evaluated by the integration of Mindlin's equation given by Douglas and Davis [45].

In the following presentation, for piles in nonhomogeneous soils, an investigation is made to propose approximate analytical solutions of the settlement for vertical loads and the lateral displacement for lateral loads. First, without using the load transfer parameter with the conventional form of adopting the maximum radius of the influence of a pile proposed by Randolph and Wroth [29] and Randolph [46], the load transfer parameter of a single pile subjected to vertical loads in nonhomogeneous soils is derived from the displacement influence factor of Mindlin's solution for an elastic continuum analysis. Second, to combine the elastic continuum approach with the subgrade reaction approach, the modulus of the subgrade reaction along the pile in nonhomogeneous soils is expressed by the displacement influence factor related to Mindlin's equation for an elastic continuum analysis. Third, the relationship between settlement and vertical load for a single pile in nonhomogeneous soils is obtained by using the recurrence equation for each layer.

By using the solutions given by Hetenyi [39] for a single pile subjected to lateral loads in nonhomogeneous soils and the modulus of the subgrade reaction represented by the displacement influence factor related to Mindlin's solution, the relationship between horizontal displacement, rotation, moment, and shear force for a single pile subjected to lateral loads in multilayered soils is obtainable in the form of the recurrence equation. For the analysis of a pile subjected to horizontal loads, a conventional assumption that a circular pile of diameter d, length L, and constant flexibility $E_P I_P$ is idealized as a thin rectangular vertical strip of width d, length L, and constant flexibility $E_P I_P$ given by Douglas and Davis [45] has been used. However, it seems that the investigation for the validity of this assumption has not been made. Therefore, a Winkler model approach proposed is adopted for a circular pile of the diameter d, length L, and constant flexibility $E_P I_P$, and a comparison is performed between the result computed from the conventional assumption and that calculated from the proposed method.

The formulation of the interaction factor between pile groups in nonhomogeneous soils is achieved using Mindlin's equation for an elastic continuum analysis. An expression for the settlement at a pile base in multilayered soils is obtainable using the equivalent elastic method [47] to generalize the equation of settlement given by Boussinesq for a homogeneous soil in the case of vertical loadings. The comparison of the results calculated by the present method for single piles and pile groups in nonhomogeneous soils subjected to vertical and lateral loads is performed with those obtained from the more rigorous FEM and BEM.

2. SETTLEMENT AND INTERACTION FACTORS OF PILES SUBJECTED TO VERTICAL LOADS IN NONHOMOGENEOUS SOILS

To obtain a solution for the values of shear stress along a pile and settlement of the pile, it is necessary to give expressions for the settlement of the pile and soil at each element in terms of the unknown shear stresses on the pile. Figure 1 shows a pile discretized into several segments of $1 \sim mb-1$ in nonhomogeneous soils, with mb denoting the $mb-$th soil layer beneath the base of the pile subjected to a vertical load. As shown in Figure 1, the present procedure uses the elastic moduli, that is, Young's modulus E_m, Poisson's ratio v_m, and thickness H_m for the mth layer in the n layers of nonhomogeneous soils; L is the length of a pile; d is the diameter of the pile shaft; P_{Pm}, P_{Sm}, and P_{Bm} are the axial force, the shear force, and the base force of the mth element, respectively; and w_{Pm} and w_{Bm} are settlements at the head and base of the mth element, respectively.

Figure 2 shows the basic geometry of a single pile for a vertical load. By making reference to equations given by Poulos and Davis [7], the settlement at a depth of the soil that is adjacent to the pile subjected to the shear stress p_S along the pile may be written as follows:

$$w = w(h) = 2d \int_0^L \int_0^{\pi/2} {}_{\mathrm{p}}I \frac{p_S}{E} \mathrm{d}\theta \mathrm{d}c \tag{1}$$

where w is the settlement of the soil, h is the depth coordinate of the node at which the settlement is evaluated, ${}_{\mathrm{p}}I$ is the influence factor for vertical displacement due to a vertical point load; E is the

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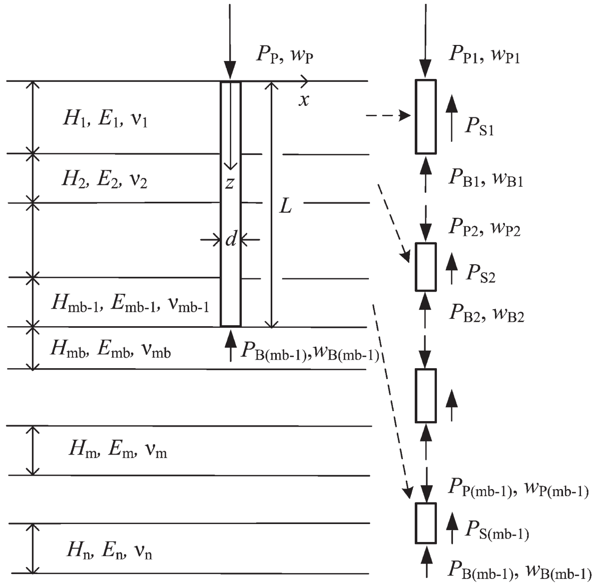

Figure 1. A pile discretized into a number of segments in multilayered soils for a vertical load.

Young's modulus of the soil, θ is the angle related to the pile section, and c is the depth coordinate of the node where the shear stress is applied.

Applying the first mean value theorem for integration [48] to Equation (1) and considering the singularity of the function of ${}_{\mathrm{p}}I$ at the depth $c=h$, the following equation may be assumed:

$$w = w(h) = dI_v(h)\frac{p_S(h)}{E(h)} \tag{2}$$

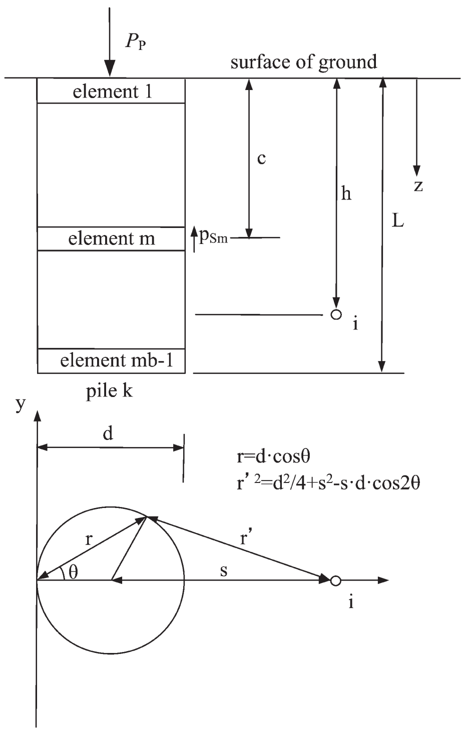

Figure 2. The basic geometry of a single pile for a vertical load.

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where $p_S(h)$ and $E(h)$ vary with depth h, and $I_v(h)$ is the settlement influence factor expressed in the following form:

$$I_v(h) = 2\int_0^L \int_0^{\pi/2} {}_{\mathrm{p}}I \mathrm{d}\theta \mathrm{d}c \tag{3}$$

where ${}_{\mathrm{p}}I$ is given by Mindlin's solution as follows:

$$\begin{aligned} {}_{\mathrm{p}}I &= \frac{(1+v)}{8\pi(1-v)}\left\{\frac{{z_1}^2}{{D_1}^3}+\frac{3-4v}{D_1}+\frac{5-12v+8v^2}{D_2}\right. \\ &\quad \left.+\frac{(3-4v){z_2}^2-2cz_2+2c^2}{{D_2}^3}+\frac{6c{z_2}^2(z_2-c)}{{D_2}^5}\right\} \\ z_1 &= h-c \\ z_2 &= h+c \\ {D_1}^2 &= r^2+{z_1}^2 \\ {D_2}^2 &= r^2+{z_2}^2 \\ r &= d\cos\theta \end{aligned} \tag{4}$$

From Equation (2), the settlement w_m of the soil adjacent to the mth element of the pile subjected to the shear stress p_{Sm} (m=1~$mb-1$) along a pile may be expressed as follows:

$$w_m = dI_{vm}\frac{p_{Sm}}{E_m} \quad (m = 1\sim mb-1) \tag{5}$$

and

$$w_m = d\zeta_m\frac{p_{Sm}}{2G_m} \quad (m = 1\sim mb-1) \tag{6}$$

where I_{vm} is the settlement influence factor related to the soil adjacent to the mth element of the pile, ζ_m is the load transfer parameter proposed by Randolph and Wroth [29] and Randolph [46], and G_m is the shear modulus of the soil adjacent to the mth element of the pile. Equation (2) implies that an integral form of the settlement influence factor is calculated for a given local field point, taking into account the characteristics that the integrand is a function of the coordinates and Poisson's ratio over the length of the pile and possesses a singularity at the local field point, and then applying this to calculation of the local settlement, taking the local values of the shear stress and Young's modulus. Therefore, Equation (2) may be available to a pile in the nonhomogeneous soil, which consists of local homogeneous soils. This approach is analogous to the load transfer method using Equation (6) given by Randolph and Wroth [29]. However, with regard to the results obtained from the rigorous approaches of the finite element and the integral equation analyses, the present method shows better agreement compared with the method of Randolph and Wroth [29]. From Equations (5) and (6), the parameters I_{vm} and ζ_m are related to each other as follows:

$$\zeta_m = I_{vm}/(1+v_m) \tag{7}$$

For the soil layers of finite depth, the settlement may be approximately obtained by using the Steinbrenner [49] approximation. Thus, the settlement w at a depth h in a layer of depth h_{RB} is written as

$$w = w(h) = dI_{v\mathrm{RB}}(h)\frac{p_S(h)}{E(h)} \tag{8}$$

where $I_{v\mathrm{RB}}(h)$ is the settlement influence factor expressed as $I_{v\mathrm{RB}}(h)=I_v(h)-I_v(h_{\mathrm{RB}})$.

For a given local field point of the soil adjacent to an element of pile, the difference between the settlement given by Poulos and Davis [7] and that expressed by Equation (5) is that the equation given

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by Poulos and Davis is represented by the sum of the settlement influence factor, shear stress, and Young's modulus at each element over the length of the pile, whereas Equation (5) is expressed by the local values of the settlement influence factor, shear stress, and Young's modulus.

For the Winkler soil model of the subgrade reaction analysis, the relationship between the shear stress p_S and the deflection w at a depth h in nonhomogeneous soils subjected to vertical loads is assumed to be related as follows:

$$p_S = k_v(h)w \tag{9}$$

where $k_v(h)$ is the modulus of the subgrade reaction and varies with depth h. The modulus of the subgrade reaction $k_v(h)$ in Equation (9) along the pile in nonhomogeneous soils is derived from the relationship between settlement and shear stress in the elastic continuum presented in Equation (2) to establish the relationship between the elastic continuum approach and the subgrade reaction approach for piles subjected to vertical loads, as follows:

$$k_v(h) = E(h)/\{dI_v(h)\} \tag{10}$$

As shown in Figure 1, in the case where the axial force P_P on the top of a pile in the first soil layer is applied, the mth element of the pile is loaded with the axial force P_{Pm} (m=1~$mb-1$) on the head, the vertical shear force P_{Sm} around the periphery, and the base force P_{Bm}. These forces satisfy the vertical equilibrium of the mth pile element; that is, P_{Pm}=P_{Sm}+P_{Bm}. It is assumed that the settlement of the pile element is identical with that of the soil element adjacent to the pile element. For the case where the head of the pile k is subjected to external load P_{P1k}, the settlement w_{P1i} of the head of the pile i is presented as

$$w_{P1i} = \sum_{k=1}^{N} F_{Pik} P_{P1k} \tag{11}$$

where N is the number of piles and

$$F_{Pik} = F_1 \quad (i = k) \tag{12}$$

$$F_{Pik} = F_1' \quad (i \neq k) \tag{13}$$

The parameter F_1 in Equation (12) is obtained by the following recurrence equation:

$$F_m = \frac{F_{m+1} + FA_m}{FB_m \times F_{m+1} + 1} \quad \left(m = 1 \sim mb - 1\right) \tag{14}$$

where

$$\begin{aligned} FA_m &= \frac{\mu_m \zeta_m}{2\pi G_m} \tanh(\mu_m H_m) \\ FB_m &= \frac{2\pi G_m}{\mu_m \zeta_m} \tanh(\mu_m H_m) \\ \lambda_m &= \frac{E_p}{G_m} \\ G_m &= \frac{E_m}{2(1+v_m)} \\ \mu_m &= \left(\frac{2}{\zeta_m \lambda_m}\right)^{1/2} \frac{2}{d} \end{aligned} \tag{15}$$

where E_P is the elastic modulus of a pile. The initial value F_{mb} in Equation (14) is obtained from the relationship between settlement and load for the pile base on multilayered soils. Considering that the pile base is represented as a rigid punch acting on the surface of soils ignoring the pile shaft and surrounding soil depth and taking into account Boussinesq's equation in a homogeneous soil, the relationship between settlement and load for the pile base on multilayered soils was obtained using the equivalent elastic method proposed by Hirai [47].

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Let us consider the formulation of the parameter F_1' represented by Equation (13). For the case where the head of the pile k is subjected to the external load P_{P1k} and the shear stress on the segment m of the shaft of the pile k caused by the load P_{P1k} is p_{Smk}, the settlement w_{Pik}' of the head of the pile i is given by Poulos and Davis [7] as follows:

$$w^{'}_{Pik} = F^{'}_{Pik} P_{P1k} \tag{16}$$

where F_{Pik}' is a parameter due to the shear stress p_{Smk}. For the case where the head of the pile k is subjected to external load P_{P1k} and the stress acting on the pile base is $p_{B(mb-1)k}$, the settlement w_{Pik}'' of the head of the pile i is obtained by Poulos and Davis [7] as follows:

$$w^{''}{}_{Pik} = F^{''}{}_{Pik} P_{P1k} \tag{17}$$

where F_{Pik}'' is a parameter due to the stress acting on the pile base $p_{B(mb-1)k}$. Therefore, for the case where the pile k (k=1~N) is subjected to the load P_{P1k}, the settlement w_{P1i} of the head of the pile i presented in Equation (11) is written by Equations (16) and (17) as follows:

$$\begin{aligned} w_{P1i} &= \sum_{k=1}^{N} \left(F^{'}_{Pik} + F^{''}_{Pik}\right) P_{P1k} \\ &= \sum_{k=1}^{N} F_{Pik} P_{P1k} \end{aligned} \tag{18}$$

where

$$F_{Pik} = F^{'}_{Pik} + F^{''}_{Pik} \tag{19}$$

It is found from Equations (13) and (19) that

$$F^{'}_{1} = F_{Pik} = F^{'}_{Pik} + F^{''}_{Pik} \quad (i \neq k) \tag{20}$$

The interaction factor α_{ik} for spacing between piles i and k is obtained from Equations (12)–(14) and (20) as follows:

$$\alpha_{ik} = F_{Pik} / F_{Pkk} = F^{'}_{1} / F_{1} \tag{21}$$

3. LATERAL DISPLACEMENT AND INTERACTION FACTORS OF PILES SUBJECTED TO LATERAL LOADS IN NONHOMOGENEOUS SOILS

Let us consider the lateral displacement for a single pile subjected to the horizontal load in a nonhomogeneous soil. Figure 3 shows a pile discretized into several segments of $1{\sim}mb-1$ in multilayered soils, with mb denoting the $mb-$th soil layer beneath the base for the pile subjected to the lateral load H_P and the moment M_P on the surface of ground. As shown in Figure 3, M_m and S_m are the moment and the shear force, respectively, at the top of the mth element, and p_m is horizontal stress along the mth element.

Figure 4 shows the basic geometry of a single pile for lateral loads. For the lateral pressure p and lateral force P, the pressure at angle θ between pile and soil caused by external loads on the pile is $p{\cdot}|\cos(2\theta)|/2$ compression on one side and $p{\cdot}|\cos(2\theta)|/2$ tension on the other side, and the lateral force per unit length of a pile is expressed as $P{=}p \times d$. By making reference to equations given by Poulos

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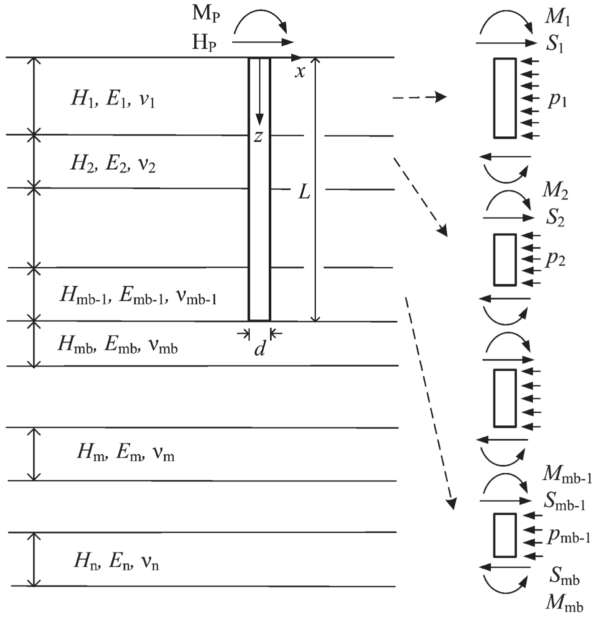

Figure 3. A pile discretized into a number of segments in multilayered soils for lateral loads.

and Davis [7], the horizontal displacement at a depth of the soil that is adjacent to the pile subjected to the lateral pressure p on the pile may be written as follows:

$$u = u(h) = d\int_0^L \int_0^{\pi/2} {}_{\mathrm{p}}I\,\frac{p}{E}|\cos 2\theta|\mathrm{d}\theta\mathrm{d}c \tag{22}$$

where u is the horizontal displacement of the soil, h is the depth coordinate of the node at which the horizontal displacement is evaluated, ${}_{\mathrm{p}}I$ is the influence factor for horizontal displacement due to a

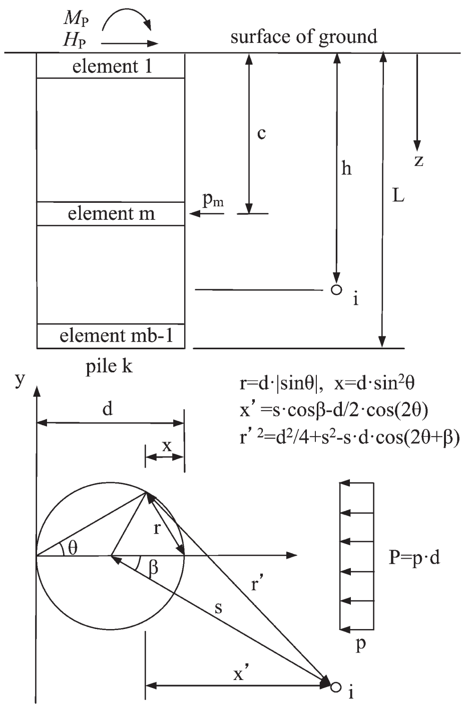

Figure 4. The basic geometry of a single pile for lateral loads.

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horizontal point load, E is the Young's modulus of the soil, θ is the angle related to the pile section, and c is the depth coordinate of the node where the horizontal stress is applied. If the lateral pressure is compression only on one side, the integral interval for the angle θ is from $-\pi/4$ to $\pi/4$ in Equation (22). Applying the first mean value theorem for integration [48] to Equation (22) and considering the singularity of the function of ${}_{\mathrm{p}}I$ at the depth $c=h$, the following equation may be assumed:

$$u = u(h) = dI_h(h)\frac{p(h)}{E(h)} \tag{23}$$

where $p(h)$ and $E(h)$ vary with depth h, and $I_{\mathrm{h}}(h)$ is the lateral displacement influence factor represented in the following form:

$$I_{\mathrm{h}}(h) = \int_0^L \int_0^{\pi/2} {}_{\mathrm{p}}I|\cos 2\theta| \mathrm{d}\theta \mathrm{d}c \tag{24}$$

where ${}_{\mathrm{p}}I$ is given by Mindlin's solution as follows:

$$
\begin{gathered}
{}_{\mathrm{p}}I = \frac{(1+v)}{8\pi(1-v)}\left[\frac{3-4v}{D_1} + \frac{1}{D_2} + \frac{x^2}{D_1^3} + \frac{(3-4v)x^2}{D_2^3} + \frac{2ch}{D_2^3}\left(1-\frac{3x^2}{D_2^2}\right) + \frac{4(1-v)(1-2v)}{D_2+h+c}\left\{1-\frac{x^2}{D_2(D_2+h+c)}\right\}\right] \\
z_1 = h - c \qquad z_2 = h + c \\
{D_1}^2 = r^2 + z_1^2 \qquad D_2^2 = r^2 + z_2^2 \\
r = d|\sin\theta| \qquad x = d\sin^2\theta
\end{gathered}
\tag{25}
$$

From Equation (23), the horizontal displacement u_m of the soil adjacent to the mth element of the pile subjected to the horizontal stress p_m on the mth element (m=1~$mb-1$) along the pile can be expressed as follows:

$$u_m = dI_{hm}\frac{p_m}{E_m} \qquad \left(m = 1 \sim mb - 1\right) \tag{26}$$

where $I_{\mathrm{h}m}$ is the lateral displacement influence factor.

Compared with the method presented by Poulos and Davis [7], which is based on the assumption that the width d of a thin rectangular vertical strip is taken as pile diameter d, the new item represented by Equation (24) is that the integration of Mindlin's equation is calculated over the actual (circular) section of the pile rather than over a thin rectangular area in which the width represents the pile diameter. Equation (23) means that an integral form of the displacement influence factor is calculated for a given local field point, taking into account the characteristics that the integrand is a function of the coordinates and Poisson's ratio over the length of the pile and possesses a singularity at the local field point, and then applying this to calculation of the local displacement, taking the local values of the lateral pressure and Young's modulus. Therefore, Equation (23) may be applicable to a pile in the nonhomogeneous soil composed of local homogeneous soils.

For the Winkler soil model of the subgrade reaction analysis, the relationship between the horizontal pressure p and the deflection u at a depth h in multilayered soils subjected to lateral loads is assumed to be related as follows:

$$p = k_h(h) \cdot u \tag{27}$$

where $k_{\mathrm{h}}(h)$ is the modulus of the subgrade reaction caused by horizontal pressure due to lateral loads and varies with the depth h.

To establish the relationship between the elastic continuum approach and the subgrade reaction approach for piles subjected to lateral loads, the modulus of the subgrade reaction $k_{\mathrm{h}}(h)$ in Equation (27) along the pile in multilayered soils is derived from the relationship between deflection u and horizontal pressure p in elastic continuum approach presented in Equation (23) as follows:

$$k_{\mathrm{h}}(h) = E(h)/\{dI_{\mathrm{h}}(h)\} \tag{28}$$

The pile subjected to lateral loads is usually assumed to be governed by the beam equation as follows:

$$E_{\mathrm{P}}I_{\mathrm{P}}\frac{d^4u}{dh^4} + k_{\mathrm{h}}(h)du = 0 \tag{29}$$

where I_{P} is the moment of inertia of the pile section. By extending the solution for a homogeneous soil given by Chang [38] to produce that for multilayered soils, the solution presented by Hetenyi [39] leads to that of Equation (29) in the following recurrence equation:

$$\begin{pmatrix} u_{m+1} \\ \theta_{m+1} \\ M_{m+1} \\ S_{m+1} \end{pmatrix} = \begin{pmatrix} F_{1m} & -\dfrac{F_{2m}}{\beta_m} & -\dfrac{F_{3m}}{E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^2} & -\dfrac{F_{4m}}{E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^3} \\ 4\beta_m F_{4m} & F_{1m} & \dfrac{F_{2m}}{E_{\mathrm{P}}I_{\mathrm{P}}\beta_m} & \dfrac{F_{3m}}{E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^2} \\ 4E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^2F_{3m} & -4E_{\mathrm{P}}I_{\mathrm{P}}\beta_m F_{4m} & F_{1m} & \dfrac{F_{2m}}{\beta_m} \\ 4E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^3F_{2m} & -4E_{\mathrm{P}}I_{\mathrm{P}}\beta_m{}^2F_{3m} & -4\beta_m F_{4m} & F_{1m} \end{pmatrix} \begin{pmatrix} u_m \\ \theta_m \\ M_m \\ S_m \end{pmatrix} \tag{30}$$

where

$$\begin{aligned} \beta_m &= \{k_{hm}d/(4E_{\mathrm{P}}I_{\mathrm{P}})\}^{1/4} \\ F_{1m} &= \cosh(\beta_m H_m) \times \cos(\beta_m H_m) \\ F_{2m} &= \{\cosh(\beta_m H_m) \times \sin(\beta_m H_m) + \sinh(\beta_m H_m) \times \cos(\beta_m H_m)\}/2 \\ F_{3m} &= \sinh(\beta_m H_m) \times \sin(\beta_m H_m)/2 \\ F_{4m} &= \{\cosh(\beta_m H_m) \times \sin(\beta_m H_m) - \sinh(\beta_m H_m) \times \cos(\beta_m H_m)\}/4 \end{aligned} \tag{31}$$

where m=1~$mb-1$; u_m, θ_m, M_m, and S_m are the displacement, the rotation, the moment, and the shear force for the pile in the mth soil layer, respectively; $u_1=u_{\mathrm{P}}$, $\theta_1=\theta_{\mathrm{P}}$, $M_1=M_{\mathrm{P}}$, and $S_1=H_{\mathrm{P}}$; u_{P} and θ_{P} are the unknown values; M_{P} and H_{P} are the specified values; and $k_{\mathrm{h}m}$ is k_{h} of the mth soil layer. In the following presentation, analysis is made for the floating pile where the boundary conditions at the tip of the pile are S_{mb}=0.0 and M_{mb}=0.0.

For a fixed-head pile subjected to a lateral load H_{P} in a semi-infinite soil with constant soil modulus, the horizontal displacement at a depth can be expressed as follows:

$$u = I_{\mathrm{uF}}\left(\frac{H_{\mathrm{P}}}{EL}\right) \tag{32}$$

where I_{uF} is the elastic influence factor for the horizontal loads on a fixed-head pile.

Let us consider the behavior of pile groups subjected to horizontal load and moment. Two identical, equally loaded piles are investigated, and each pile is divided into several elements for the single pile. As shown in Figure 4, s is the center-to-center pile spacing, and β is the angle between the line joining the pile centers and the loading direction, which is termed the *departure angle*. For the case where the pile k is subjected to the horizontal load H_{P} and moment $\mathrm{M_p}$ on the top of the pile and the horizontal stress on the segment m of the shaft of the pile k caused by H_{P} and M_{P} is p_{mk}, the horizontal displacement u_{ik} of the head of the pile i may be given as follows:

$$u_{ik} = d\int_0^L \int_{-\pi/2}^{\pi/2} {}_pI\frac{p}{2E}|\cos 2\theta|\mathrm{d}\theta\mathrm{d}c = d\sum_{m=1}^{mb-1}\frac{I_{imk}}{2E_{mk}}\cdot p_{mk} \tag{33}$$

where E_{mk} is the Young's modulus of the soil adjacent to the mth element of the pile k and

$$I_{imk} = \int_{TH_{m-1}}^{TH_m} \int_{-\pi/2}^{\pi/2} pI|\cos 2\theta| \mathrm{d}\theta \mathrm{d}c \tag{34}$$

where $TH_m = \sum_{i=1}^{m} H_i$.

For interaction factors α_{uH} and α_{uF} denoted for free-head and fixed-head piles, respectively, with the spacing between piles i and k, the interaction factor α_{ik} between piles i and k is obtained from Equations (30) and (33) in the following form:

$$\alpha_{ik} = u_{ik}/u_{kk} \tag{35}$$

where u_{kk} represents the appropriate unit-reference displacement, that is, the displacement of a single pile under unit horizontal load. The displacement u_{kk} in Equation (35) is presented in Equation (26) at the head of the pile k and is calculated by the recurrence formula presented in Equation (30).

The group displacement may be expressed as a group reduction factor R_R, which is defined as the ratio of the group displacement to the displacement of a single pile carrying the same total load as the group. Then, the group reduction factor R_R is calculated as follows:

$$R_R = u_G/(H_G u_{kk}) \tag{36}$$

where u_G is the group displacement and H_G is the total load on the pile group.

4. ANALYSIS OF VERTICALLY AND LATERALLY LOADED PILES IN NONHOMOGENEOUS SOILS

Let us consider the settlement of a single pile subjected to vertical load on the surface of a nonhomogeneous soil. It is of interest that the load transfer parameters ζ_m in Equation (6) can be determined using the settlement influence factor I_{vm} without the use of the conventional form ζ_m=ln $(2r_m/d)$ of adopting the maximum radius of the influence of the pile, r_m. For a nonhomogeneous soil, the maximum radius of the influence of the pile is defined as $r_m=2.5\rho(1-\nu_S)L$, where ρ is a nonhomogeneity factor, which is the ratio of the shear modulus at the pile mid-depth to that at the base, that is, ρ=G(L/2)/G(L), and ν_S is Poisson's ratio. Figure 5 illustrates relationships among the depth ratio z/L, the load transfer parameter $\zeta_m=\ln(2r_m/d)$ given by Randolph and Wroth [29], and the load transfer parameter ζ_m* defined by Equation (6) for a homogeneous soil having $\nu_S=0.5$ and $\rho=1$. The difference between the load transfer parameters ζ_m and ζ_m* is that the load transfer parameter ζ_m takes a constant value dependent on the slenderness ratio of pile L/d and the Poisson's ratio of the soil, whereas the load transfer parameter ζ_m* depends on the z/L ratio, the L/d ratio, and the Poisson's ratio of the soil.

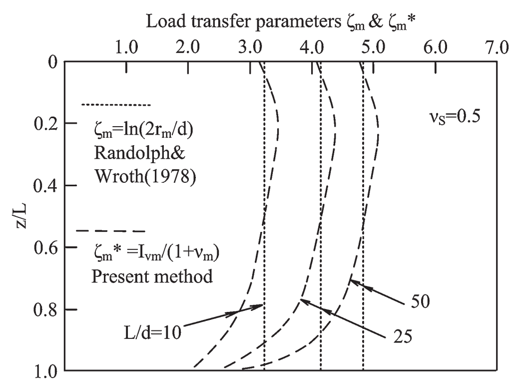

Figure 5. Relationships between z/L and load transfer parameters.

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Figure 6 shows the distribution of the shear stress down a pile surface for a rigid pile of the slenderness ratio L/r_0=40 and the radius $r_0=d/2$ and a homogeneous soil having the Poisson's ratio ν_S=0.4. The variables are $p_S(z)$, the shear stress on the pile shaft at a depth z, and G_S, the shear modulus of the soil. As shown in Figure 6, Equation (8) in the article given by Randolph and Wroth [29] takes a constant value along the pile shaft. The result obtained from the present method is compared with that from the finite element analysis and integral equation analysis given by Randolph and Wroth [29] and shows good agreement, except for the neighborhood of the pile top.

The value of Poisson's ratio ν_S of a soil affects the value of settlement. Figure 7 shows the settlement profile at the mid-depth of a pile having L/r_0=40 for a homogeneous soil in the case of ν_S= 0.0 and 0.5. The settlement Equation (11) in the article given by Randolph and Wroth [29] is in good agreement with the integral equation analysis, except for the range of the large radius in the case of ν_S=0.5. Results obtained from the present method are compared with those from the integral equation analysis given by Randolph and Wroth [29] and show good agreement.

It is now assumed that the stiffness increases proportionally or linearly with depth as in a Gibson soil with the nonhomogeneity factor ρ=0.5. The behavior of a rigid pile in such a soil will be investigated. As depicted in Figures 8 and 9 for the case of a pile of slenderness ratio 40 and a Gibson soil having ν_S=0.4, the shear strain and the shear stress obtained from Equations (8) and (10), respectively, in the article given by Randolph and Wroth [29] take a constant value along the pile shaft and increase linearly with depth respectively. Compared with results from the finite element analysis given by Randolph and Wroth [29], the result of the shear strain obtained from the present method shows good agreement, except for the neighborhood of the pile top, and good agreement is observed for the result of the shear stress along the pile shaft.

Figure 10 shows the comparison of settlement at the mid-depth of a pile of slenderness ratio 40, for both a homogeneous soil and a Gibson soil having ν_S=0.4. The degree of the nonhomogeneity of a

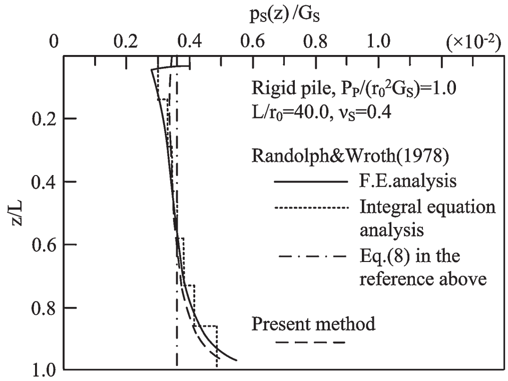

Figure 6. Distribution of shear stress down pile surface for a homogeneous soil.

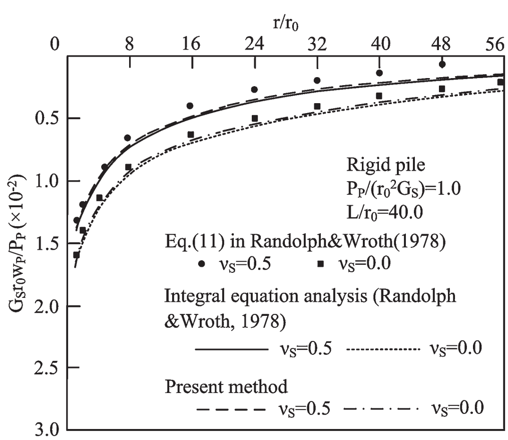

Figure 7. Settlement profile at mid-depth of pile for a homogeneous soil.

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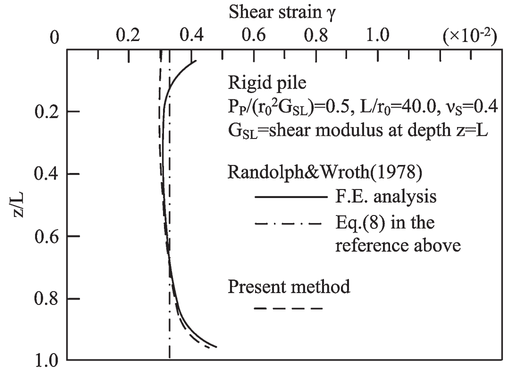

Figure 8. Distribution of shear strain down pile surface in a Gibson soil.

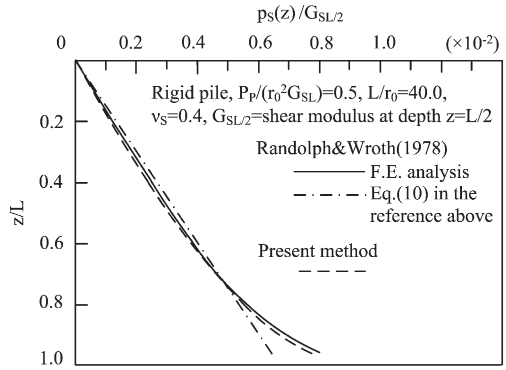

Figure 9. Distribution of shear stress down pile surface in a Gibson soil.

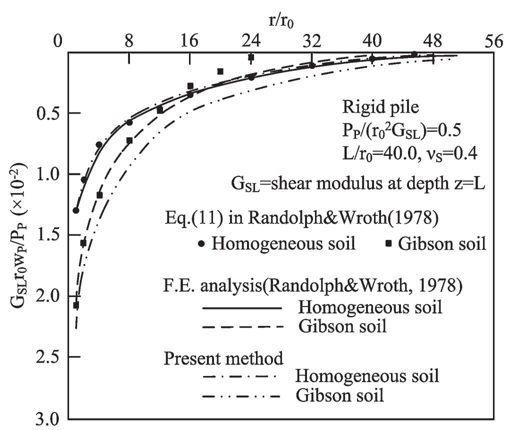

Figure 10. Comparison of settlement at the mid-depth of the pile for a homogeneous soil and a Gibson soil.

soil affects the value of settlement. Equation (11) in the article given by Randolph and Wroth [29] enables the prediction of the settlement of a homogeneous soil and that of a Gibson soil, except for the range of the large radius in the case of a Gibson soil. The present method gives good agreement for overall value of radius in comparison with the finite element analysis obtained from Randolph and Wroth [29].

For the stiffness coefficient [50] generally used in elastodynamics, the vertical stiffness coefficient of a soil around a pile is written as $K_v = P_S / w$, where $P_S = \pi p_S d$ is the vertical shear force per unit length of a pile and w is the settlement of a soil around a pile. Thus, the vertical stiffness coefficient $K_v = \pi E_S / \{(1+\nu_S)\zeta\}$ is obtained from Equation (6), and the load transfer parameter ζ given by Randolph and Wroth [29] and the vertical stiffness coefficient $K_{vA} = \pi E_S / I_v(h)$ is obtained from Equation (2), with E_S as Young's modulus of the soil.

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Figure 11 illustrates relationships between the depth ratio *z*/*L* and the vertical stiffness coefficients K_v and K_{vA} for a homogeneous soil having ν_S=0.5. The difference between the vertical stiffness coefficients K_v and K_{vA} is that K_v takes a constant value dependent on the slenderness ratio of pile *L*/*d* whereas K_{vA} depends on both *z*/*L* and *L*/*d*. Further, it is found that the value at the mid-depth of the vertical stiffness coefficient K_{vA} gives good approximation of K_v.

For an end-bearing pile where the tip of the pile bears on to a stratum that is stiffer than the soil along the shaft of the pile, Figure 12 shows relationships between the E_b/E_S and the base modulus correction factor R_b given by Poulos and Davis [7]. The settlement of the pile top and the settlement influence factor for the incompressible pile in semi-infinite mass are denoted as w_P and I_0, respectively. The parameter R_k is the correction factor for pile compressibility, which is dependent on the ratio of length to diameter *L*/*d* and the ratio of the Young's modulus of pile to soil *K*, and is defined by Poulos and Davis [7]. As the ratio of the Young's modulus of base to soil, E_b/E_S, increases, R_b decreases pronouncedly with the increase of *K*. It is found that the present method is able to represent properly the result calculated by Poulos and Davis [7].

In the following presentation, the nonhomogeneity index is defined as $\eta = E_{S0}/E_{SL}$, where E_{S0} and E_{SL} are the Young's moduli for z=0 and z=*L*, respectively. For the three kinds of η (0.0, 0.5, and 1.0) in nonhomogeneous soils where the elastic modulus increases linearly with depth, Figure 13 exhibits relationships between the ratio of the Young's modulus of pile to soil, $K_b = E_P/E_{SL}$, and the settlement influence factor I_w. It is seen that the settlement influence factor I_w decreases with increase of the nonhomogeneity index η as K_b increases. The comparison between the results presented by Poulos [5] and those computed by the present method shows good agreement.

Figure 14 illustrates relationships between the depth and the shear stress $p_S \pi d L / P_P$ for the single pile in soils that take three kinds of the nonhomogeneity index η (0.0, 0.5, and 1.0). It is seen that the present method describes properly the shapes of distribution of shear stress given by Poulos [5].

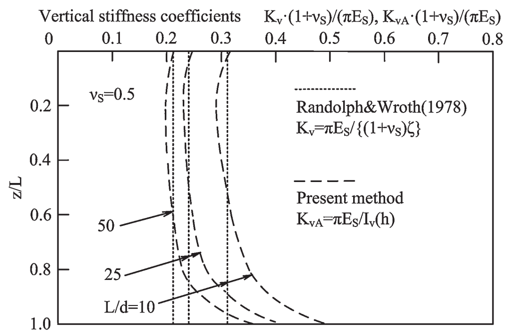

Figure 11. Relationships between *z*/*L* and vertical stiffness coefficients.

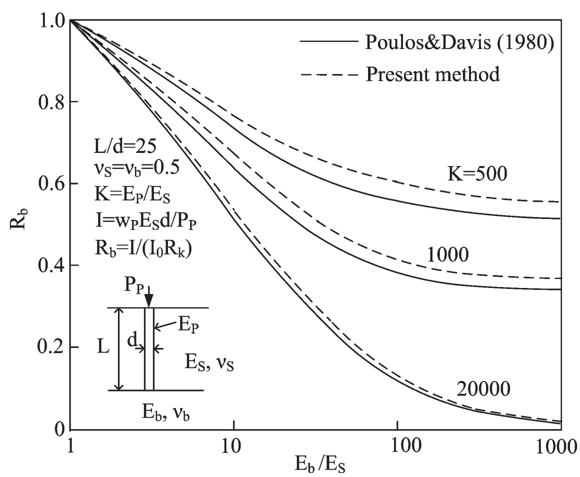

Figure 12. Relationships between E_b/E_S and the base modulus correction factor R_b.

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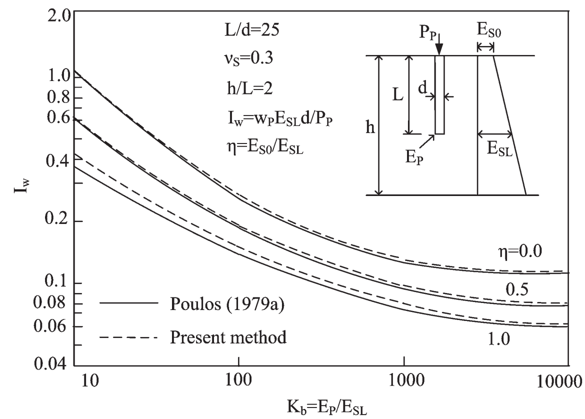

Figure 13. Relationships between the K_b and the settlement influence factor I_w.

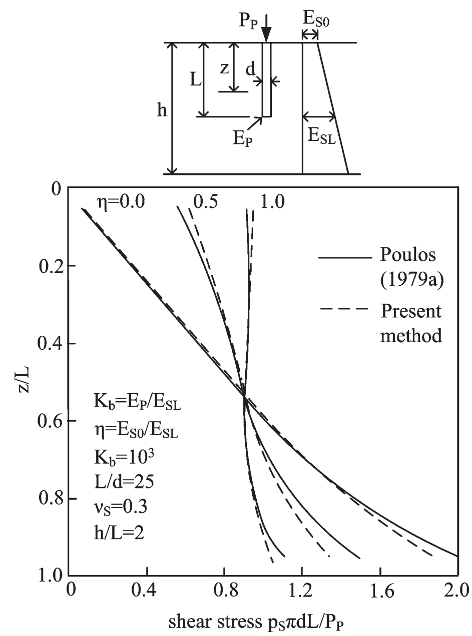

Figure 14. Relationships between the z/L and the shear stress $p_S\pi dL/P_P$.

Figures 15(a) and 15(b) show a single pile embedded in the three idealized cases of layered soils and a single pile embedded in the three cases with equivalent elastic modulus of layered soils, respectively. The means of obtaining an equivalent elastic modulus of the soil along the shaft is proposed by Hirai [47], and the values of equivalent elastic modulus of the soil are presented in Figure 15(b). Various solutions obtained for the settlement of the pile head are given in Table I. The results calculated from the present method are compared with those from the finite element and boundary element approaches by Poulos [5]. There is reasonable agreement between the solutions for cases 1 and 3. For case 2, however, the difference between the solutions of the settlement influence factor is fairly conspicuous. It has been reported by Chow [10], Lee [11], and Kitiyodom and Matsumoto [17] that the difference between calculated results has been caused by the analytical approach used and the type of layered soils. Also, it is presented by El Sharnouby and Novak [51], Southcott and Small [52], and Mylonakis [37] that the differences between these different methods may be considered to be due to the number of elements used, which is influenced by the stiffness ratio E_P/E_s and the length–diameter ratio L/d.

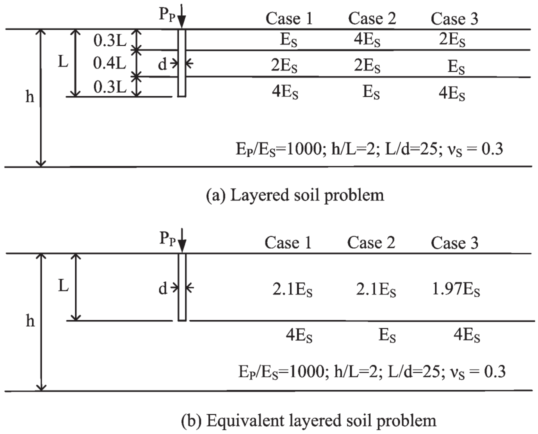

Figure 15. Layered soil problems analyzed (a single pile).

Table I. Comparison between various solutions for settlement of a single pile in layered soils.

Case	Settlement influence factor (I_w)				
	BEM [5]	BEM, equivalent modulus [5]	FEM [5]	Present method	Present method, equivalent modulus
1	0.0386	0.0381	0.0377	0.0399	0.0387
2	0.0330	0.0706	0.0430	0.0354	0.0405
3	0.0366	0.0391	0.0382	0.0392	0.0402

$I_w = w_P E_S d / P_P$; w_P=settlement of the pile; E_S=Young's modulus of soil; d=pile diameter; P_P=applied vertical load.

Figure 16 shows relationships between the ratio of the spacing of piles to pile–diameter s/d and interaction factor α_F for a group of two floating piles in a semi-infinite homogeneous soil. It is found that the interaction factor decreases with the decrease of K as s/d increases. The interaction factors computed by the present method are in good agreement with those presented by Poulos and Davis [7] and Mylonakis and Gazetas [36].

Figure 17 illustrates relationships between the s/d and the interaction factor α_F of a group of two piles to investigate the influence of the ratio of the Young's modulus of pile to soil, E_P/E_{SL}, and the pile slenderness ratio, L/d, on piles embedded in Gibson soil with the nonhomogeneity index η=0.0 with a finite layer depth. The results computed by the present method are compared with those obtained from the BEM presented by Banerjee [4] and the FEM presented by Chow [10]. It is seen that the interaction factor tends to decrease with the decrease of E_P/E_{SL} as s/d increases, and there is reasonable agreement between these computed results.

For the interaction between piles in multilayered soils, Figure 18 shows relationships between s/d and interaction factor α_F of a group of two piles for four layered soil cases with a finite layer depth. The results computed by the present method are compared with those obtained from the FEM and BEM given by Chow [9], a simplified method by Kitiyodom and Matsumoto [17], and the procedure presented by Mylonakis and Gazetas [36]. It is seen from the results of the FEM that the values of the interaction factor tend to be larger in the case where the Young's moduli of upper layers are larger than those of lower layers. However, on the basis of the results calculated by the BEM [17] and the present method, the values of the interaction factor tend to not change pronouncedly according to the type of layered soils. The results by Mylonakis and Gazetas [36] tend to be between those by the FEM and BEM for cases 3 and 4. Although there is good agreement between solutions for cases 1 and 2, the difference between solutions for cases 3 and 4 is fairly pronounced. It has been reported by Chow [10], Lee [11], and Kitiyodom and Matsumoto [17] that the calculated results have been affected by the analytical approach used and the type of layered soils. Also, as mentioned in Figure 15 and Table I, it

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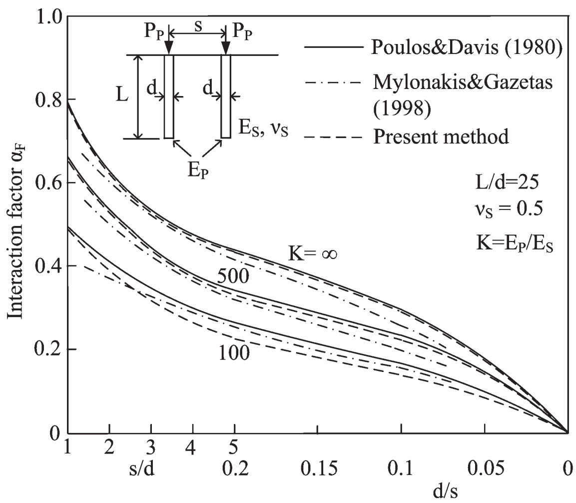

Figure 16. Relationships between the *s/d* and the interaction factor α_F.

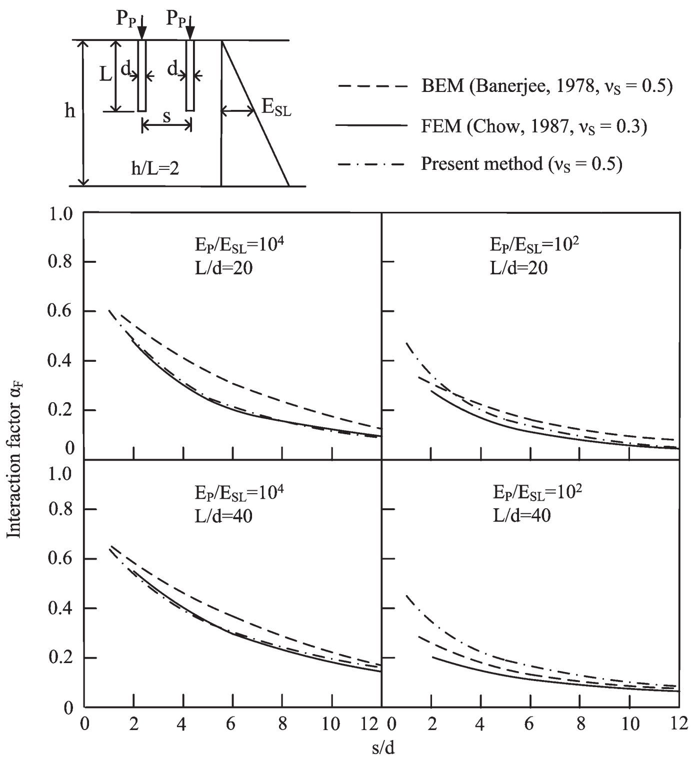

Figure 17. Relationships between the *s/d* and the interaction factor α_F.

is considered that the differences between these different methods may be due to the number of elements used, which is influenced by the stiffness ratio E_P/E_s and the length–diameter ratio *L/d*.

For the load displacement behavior due to interaction between a large number of piles in a homogeneous soil, Figure 19 illustrates relationships between *L/d* and $P_P/(G_S w_P d)$. The parameter λ is defined as $\lambda = E_P/G_S$. Butterfield and Banerjee [1] used the BEM to obtain a rigorous solution based on Mindlin's equation for axially loaded compressible pile groups with floating caps in a semi-infinite homogeneous soil. It is found from Figure 19 that a fairly good agreement is observed between the results computed by Butterfield and Banerjee [1] and those calculated by the present method.

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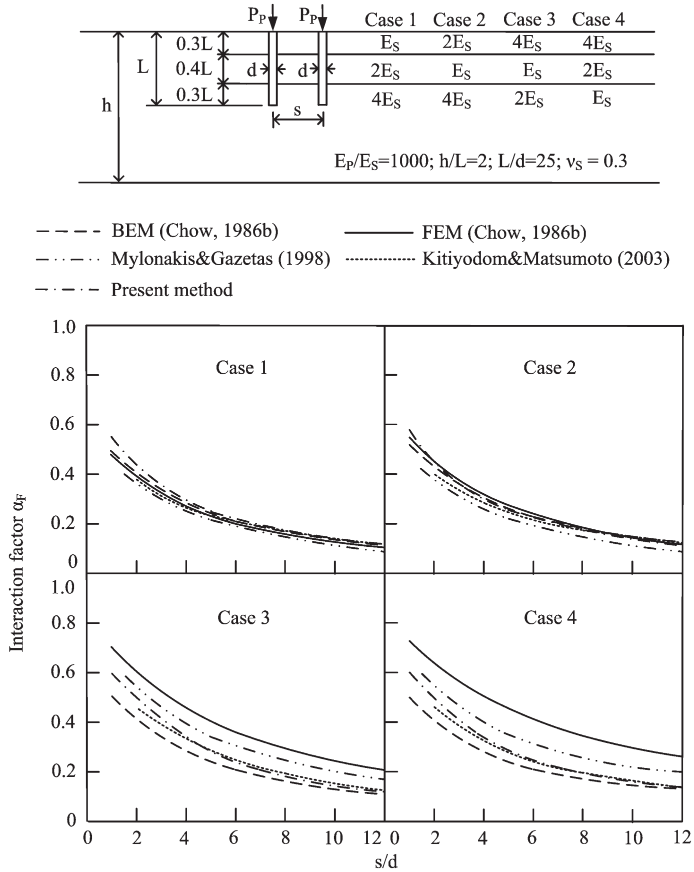

Figure 18. Relationships between the s/d and the interaction factor α_F.

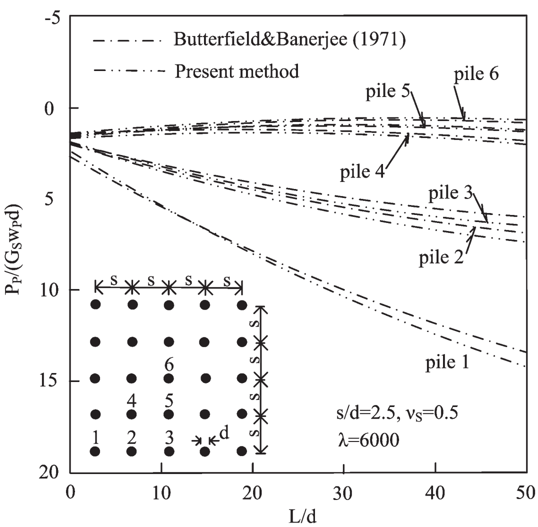

Figure 19. Relationships between L/d and $P_P/(G_S w_P d)$.

The lateral stiffness coefficient of a soil around a pile is written as $K_h = P/u$, where $P = p \times d$ is the lateral force per unit length of a pile and u is the lateral displacement of a soil around a pile. Thus, the lateral stiffness coefficient $K_{hA} = E_S/I_h(h)$ is obtained from Equation (23).

Figure 20 illustrates relationships between the depth ratio z/L and the lateral stiffness coefficients K_h for a homogeneous soil having ν_S=0.5. Using the method where the solutions of the elastic continuum

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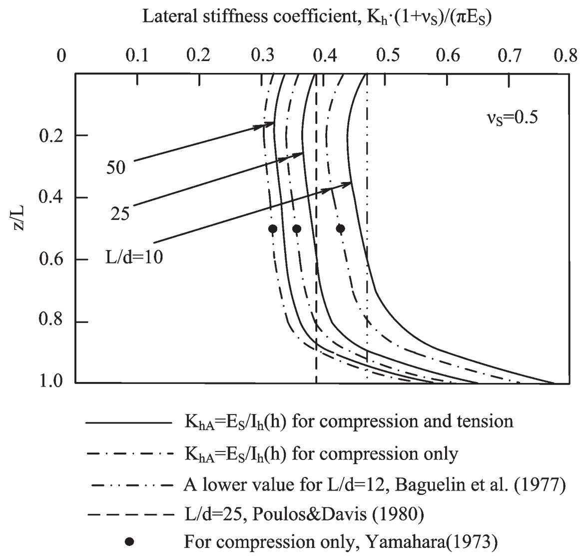

Figure 20. Relationships between *z/L* and lateral stiffness coefficients.

approach and those of the subgrade reaction approach for the displacement of a stiff fixed-head pile subjected to lateral loads are equated, Poulos and Davis [7] presented a relationship between the lateral stiffness coefficient K_h and the Young's modulus E_s given by $K_h=0.82E_s$ for the case of $L/d=25$ and Poisson's ratio $\nu_S=0.5$. Baguelin *et al.* [35] presented theoretically that the lower value of the stiffness coefficient is $K_h=0.98E_s$ for the case of $L/d=12$ and Poisson's ratio $\nu_S=0.5$ and suggested that K_h may be simply taken as being equal to E_s for the practical calculation. The lateral stiffness coefficients proposed are presented for two cases, where one is that the lateral pressure between pile and soil is compression on one side and tension on the other side and the other is that the lateral pressure is compression only on one side. It is found that the lateral stiffness coefficient K_h proposed in this study depends on z/L whereas those presented by Poulos and Davis [7] and Baguelin *et al.* [35] are independent of z/L. However, there is good agreement between the average values of the lateral stiffness coefficients for the case where the lateral pressure is compression on one side and tension on the other side. Using Mindlin's solution for lateral loads, Yamahara [42] investigated the case where the lateral pressure is compression only on one side and presented the lateral stiffness coefficients analytically at the mid-depth of a pile with circular section in a homogeneous soil. At the mid-depth of the pile, the lateral stiffness coefficients proposed for the case where the lateral pressure is compression only on one side coincide with those given by Yamahara [42].

Adopting the *Recommendations for Design of Building Foundations* [53] published by the Architectural Institute of Japan, which will be called *recommendations* for brevity, the lateral stiffness coefficient is written as $K_{hR}=0.0316\alpha E_S d^{1/4}$, where $\alpha=60$ and 80 for cohesive soils and sandy soils, respectively, and the influence of Poisson's ratio of a soil on the lateral behavior is disregarded.

Figure 21 shows relationships between the depth ratio z/L and the lateral stiffness coefficients K_{hR} and K_{hA} for a homogeneous soil having $\nu_S=0.5$. As the diameter of a pile increases, the lateral stiffness coefficient given by the recommendations becomes larger than those obtained from Yamahara [42] and the present method.

Figure 22 shows relationships between the diameter d and the K_{hR}/K_{hA} ratio for lateral stiffness coefficients at the mid-depth of a pile. As the diameter of a pile and L/d increase, the lateral stiffness coefficient given by the recommendations becomes larger than those obtained from Yamahara [42] and the present method, and this tendency is more remarkable for sandy soils of $\alpha=80$ than it is for cohesive soils of $\alpha=60$.

For a floating pile in a semi-infinite soil with increasing soil modulus, the relative pile flexibility ratio K_R is defined as $K_R=E_P I_P / (E_{SL} L^4)$. The influence of distribution of elastic modulus, which is related to the nonhomogeneity of a soil, on the calculated results is explored. For the nonhomogeneity

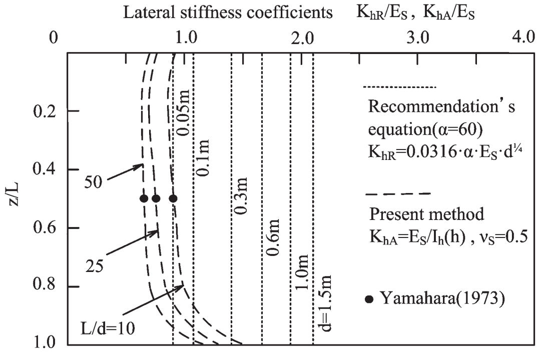

Figure 21. Relationships between *z/L* and lateral stiffness coefficients.

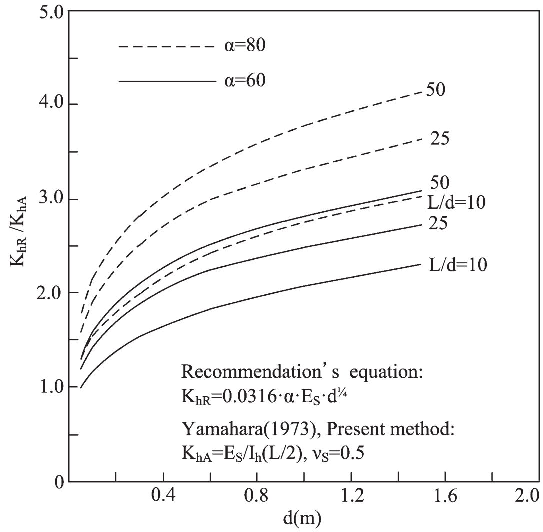

Figure 22. Relationships between the diameter *d* and the K_{hR}/K_{hA} ratio for lateral stiffness coefficients.

index η of a soil, the investigation will be made for homogeneous soils (η=1.0), moderately nonhomogeneous soils (η=0.5), and highly nonhomogeneous soils (η=0.0).

Figure 23 illustrates relationships between the depth ratio *z/L* and the nondimensional bending moment $M(z)/H_PL$ due to the lateral load applied to the free head of rigid and flexible piles embedded in nonhomogeneous and homogeneous soils (η=0.0, 0.5 and 1.0). It is found that the maximum bending moment increases with decrease of the nonhomogeneity index η and increase of the flexibility ratio K_R. The comparison between the results simulated by Banerjee and Davies [3] and those obtained from the present method shows fairly good agreement.

Figure 24 exhibits relationships between the depth ratio *z/L* and the nondimensional bending moment $M(z)/M_P$ due to an applied moment M_P at the free head of rigid and flexible piles embedded in nonhomogeneous and homogeneous soils (η=0.0 and 1.0). It is seen that the bending moment decays more markedly with the depth as the nonhomogeneity index η increases and the flexibility ratio K_R decreases. The comparison between the results simulated by Banerjee and Davies [3] and those obtained from the present method provides very close agreement.

Figure 25 illustrates relationships between the depth ratio *z/L* and the nondimensional bending moment $M(z)/H_PL$ due to the lateral load for a fixed head of rigid and flexible piles embedded in homogeneous and nonhomogeneous soils (η=1.0 and 0.0). It is seen that the bending moment becomes smaller with the depth as the nonhomogeneity index η increases and the flexibility ratio K_R decreases. The comparison between the results presented by Banerjee and Davies [3] and those obtained from the present method gives reasonable agreement.

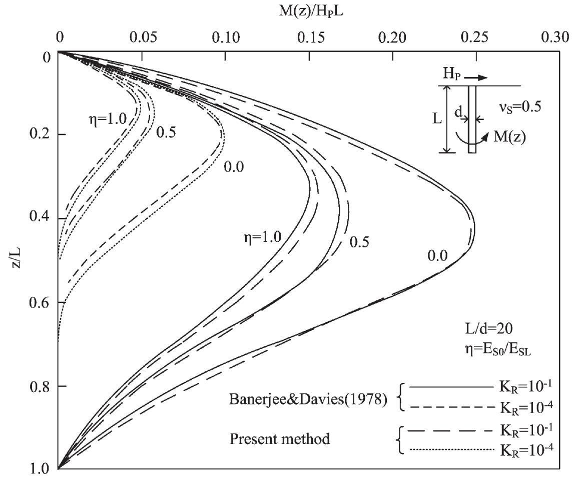

Figure 23. Relationships between *z*/*L* and moment for a free-head pile in homogeneous and nonhomogeneous soils due to lateral load.

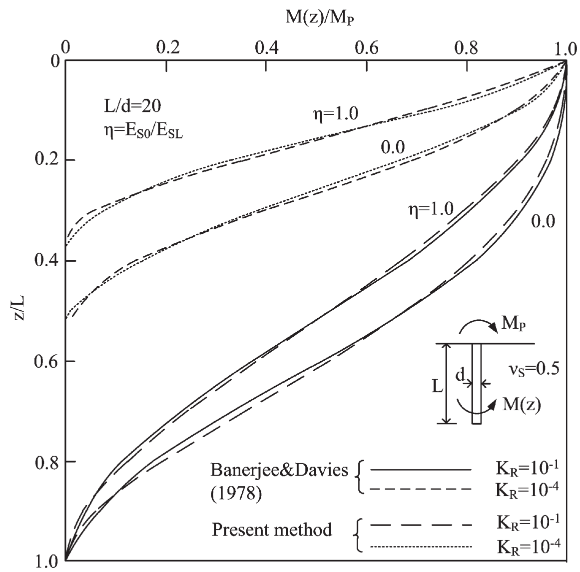

Figure 24. Relationships between *z*/*L* and moment for a free-head pile in homogeneous and nonhomogeneous soils due to applied moment.

Figure 26 exhibits relationships between the K_R and the elastic influence factor of a fixed-head pile. For homogeneous and nonhomogeneous soils, the comparison between the results given by Banerjee and Davies [3] and those obtained by the present method provides good agreement.

Figure 27 exhibits relationships between the nonhomogeneity index η and the lateral stiffness $H_P/(E_{SL}ud)$ for free-head and fixed-head piles with $L/d=20$. The comparison between the results given by Banerjee and Davies [3] and those obtained by the present method provides general similarity in respect that the lateral stiffness is dependent on the value of the nonhomogeneity index over the entire range of relative pile flexibility ratios.

Let us consider the interaction between two piles embedded in a semi-infinite homogeneous soil. Figures 28 and 29 show relationships between the center-to-center pile–spacing ratio s/d and the interaction factor α_{uH} due to the lateral load for free-head piles with values of $L/d=10$, 25, and 100. Figures 30 and 31 illustrate relationships between the center-to-center pile–spacing ratio s/d and the interaction factor α_{uF} due to the lateral load for fixed-head piles with values of $L/d=10$, 25, and 100.

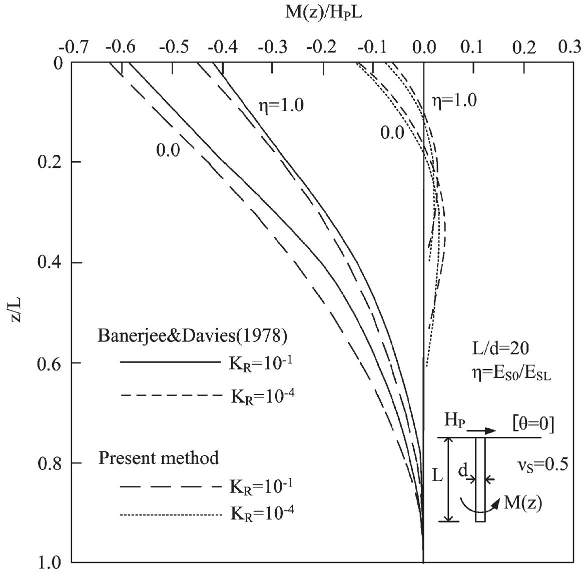

Figure 25. Relationships between *z*/*L* and moment for a fixed-head pile in homogeneous and nonhomogeneous soils due to lateral load.

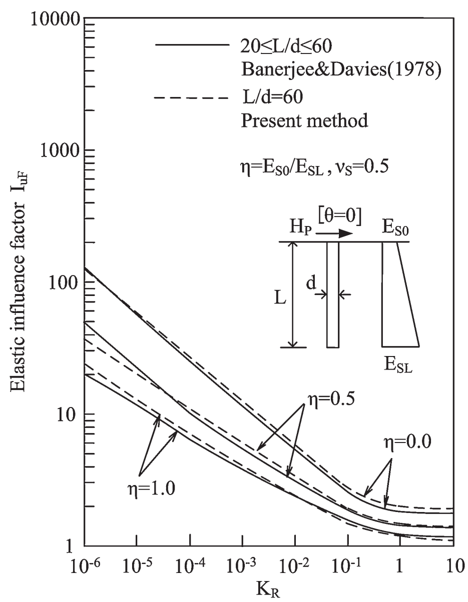

Figure 26. Relationships between the K_R and the elastic influence factor I_{uF} for a fixed-head pile in homogeneous and nonhomogeneous soils.

For the value of flexibility ratio $K_R = 10^{-5}$, two cases of $\beta = 0°$ and 90º of departure angle are investigated.

Figures 28–31 show that (1) the interaction factors α_{uH} and α_{uF} decrease with increasing spacing and are greater for $\beta = 0°$ than for $\beta = 90°$; (2) the interaction factors increase with increasing *L*/*d*; and (3) as the spacing ratio approaches $s/d = 1$, the interaction factors obtained by both El Sharnouby and Novak [54] and the present method become smaller than those presented by Poulos [27]. The calculation performed by Poulos [27] is based on the assumption that the width *d* of a thin rectangular vertical strip is taken as pile diameter *d*; however, the calculations carried out by El Sharnouby and Novak [54] and the present method assume piles of circular section. El Sharnouby and Novak [51], Southcott and Small [52], and El Sharnouby and Novak [54] suggested that Poulos' interaction factors do tend to overestimate the interaction of piles because of the small number of elements used and

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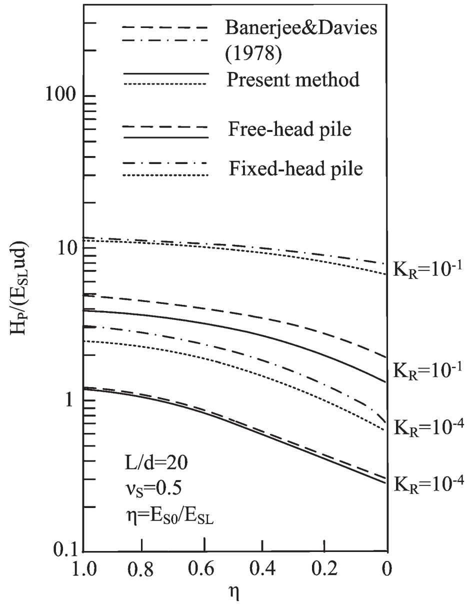

Figure 27. Relationships between η and lateral stiffness for a single pile in homogeneous and nonhomogeneous soils.

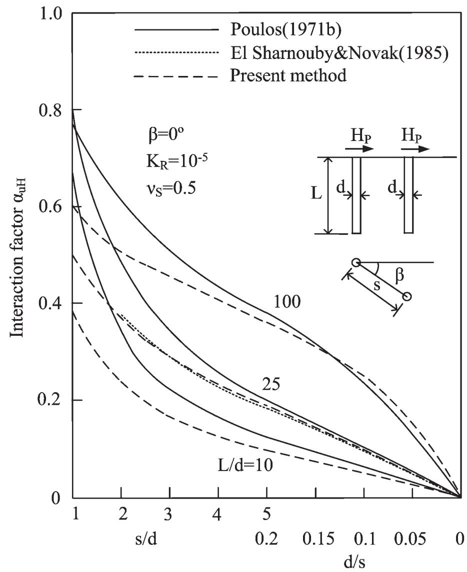

Figure 28. Relationships between the *s*/*d* and the interaction factor α_{uH}.

differences between these different methods are likely to be due to both the number of elements used and the pattern of loading of the soil continuum.

Figures 32(a) and 32(b) show two piles embedded in the four idealized cases of layered soils and two piles embedded in the four cases with equivalent elastic moduli of layered soils, respectively. The means of obtaining an equivalent elastic modulus of the soil along the shaft is proposed by Hirai [47], and the values of equivalent elastic modulus of the soil are presented in Figure 32(b). For the interaction between piles in multilayered soils subjected to lateral loads, Figure 33 shows relationships between *s*/*d* and interaction factor α_{uH} for two piles for four different layered soil cases with a finite layer depth, as shown in Figures 32(a) and 32(b). The results computed by the present method are compared with those given by Kitiyodom and Matsumoto [17]. It is found that although the interaction factor decreases with the increase in the ratio of spacing to diameter *s*/*d* for four cases, the values of the

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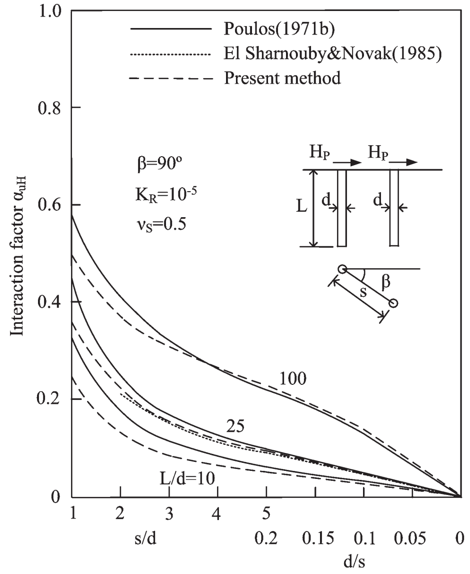

Figure 29. Relationships between the *s*/*d* and the interaction factor α_{uH}.

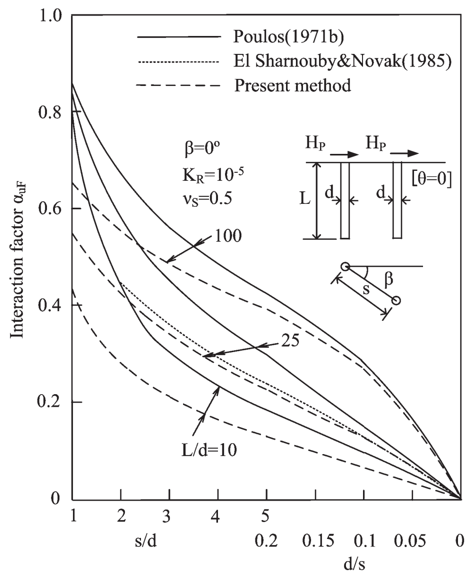

Figure 30. Relationships between the *s*/*d* and the interaction factor α_{uF}.

interaction factor tend to not change conspicuously according to the type of layered soils. The comparison between the results computed by Kitiyodom and Matsumoto [17] and those obtained from the present method shows good agreement overall.

Let us investigate the case of general pile groups subjected to lateral loads. For comparison between the free-head and the fixed-head piles of the square groups of 2^2, 3^2, and 4^2 in a homogeneous soil, Figure 34 exhibits relationships between *s*/*d* and group reduction factors R_{RuH} and R_{RuF} for free-head and fixed-head groups where each pile displaces equally. It is seen that the R_{RuH} for a group of free-head piles is smaller than the R_{RuF} for a group of fixed-head piles, and the values of R_{RuH} and R_{RuF} obtained by the present method generally tend to be smaller than those presented by Poulos [27]. The calculation performed by Poulos [27] is based on the assumption that the width *d* of a thin rectangular vertical strip is taken as pile diameter *d*; however, the calculation carried out by the present method assumes a pile of circular section. El Sharnouby and Novak [51], Southcott and Small [52], and El Sharnouby and Novak [54] suggested that Poulos' interaction factors do tend to overestimate the

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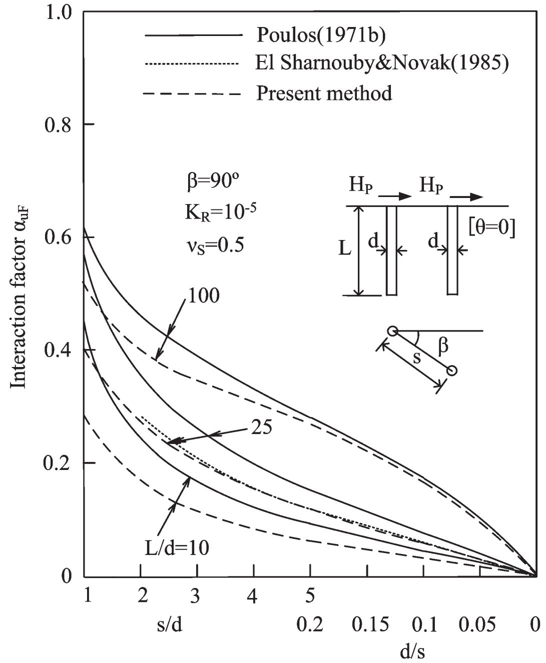

Figure 31. Relationships between the *s*/*d* and the interaction factor α_{uF}.

interaction of piles because of the small number of elements used. Therefore, it may be considered that the difference between the results given by Poulos [27] and those obtained from the present method is due to the shape of pile section and the number of elements used to represent the pile.

For groups situated in a homogeneous soil and in which all pile heads are fixed and displace equally, the horizontal pile head loads for each pile within 4^2 groups are shown in Figure 35. The parameters H and $H_{av}=H_G/N$ imply the horizontal load distributed to a pile and the average horizontal load for N piles, respectively. It is found that the outer piles carry the greatest load and the center piles have the least, and the distribution of the horizontal load becomes uniform as the spacing increases. The results obtained from El Sharnouby and Novak [54], Zhang and Small [15], and the present method indicate that for piles with the spacing ratio of less than 6, there is a poor agreement of the horizontal load distributions in comparison with the results given by Poulos [27]. Also, the results obtained from the present method are in considerably good agreement with those given by

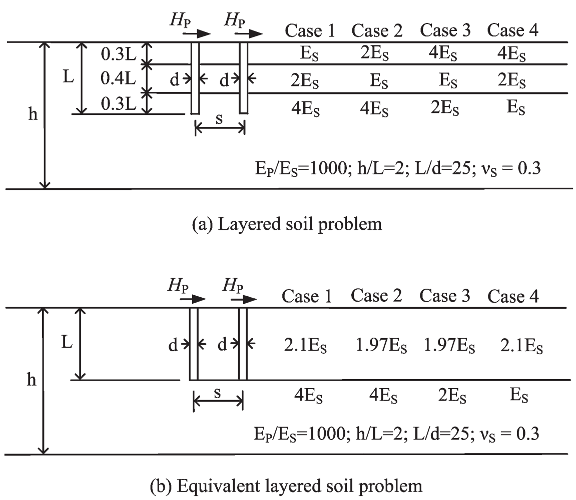

Figure 32. Layered soil problems analyzed (two piles).

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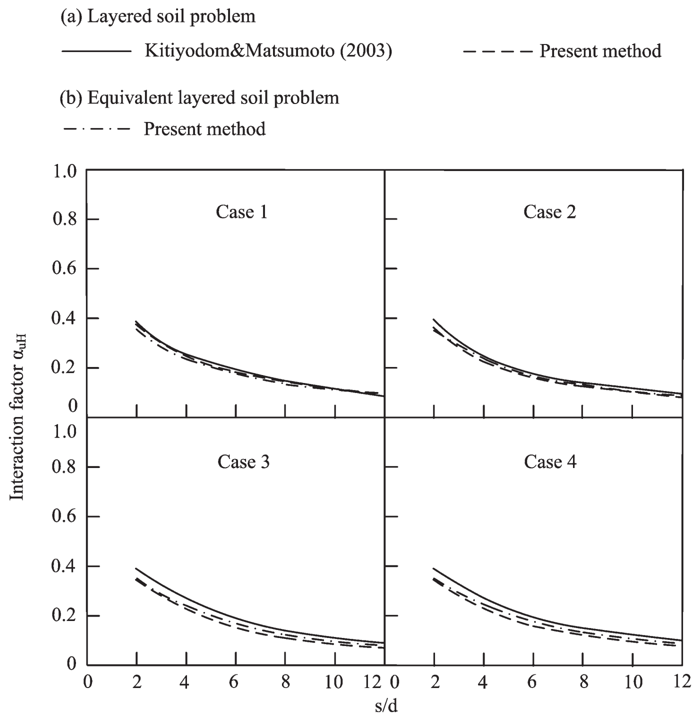

Figure 33. Relationships between the *s*/*d* and the interaction factor α_{uH}.

El Sharnouby *et al*. and Zhang *et al*. It may be considered that this is because the calculation performed by Poulos [27] is based on the assumption that the width *d* of a thin rectangular vertical strip is taken as pile diameter *d*; however, calculations carried out by other methods assume piles of circular section. Also, it is suggested by El Sharnouby and Novak [51], Southcott and Small [52], and El Sharnouby and Novak [54] that the results obtained by Poulos tend to give higher interaction factors, presumably because of the small number of elements used to represent the pile.

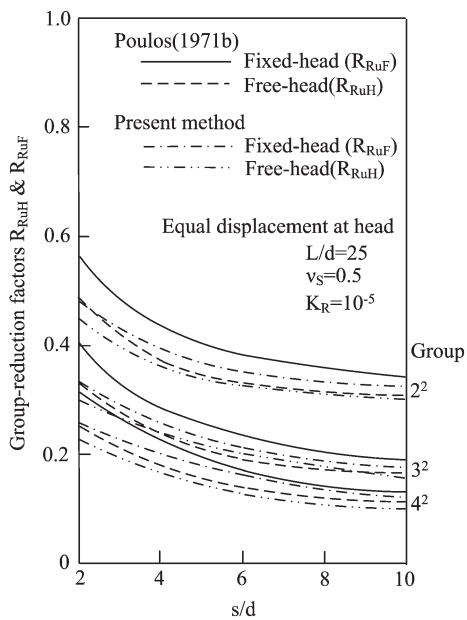

Figure 34. Relationships between the *s*/*d* and the group reduction factors R_{RuH} and R_{RuF}.

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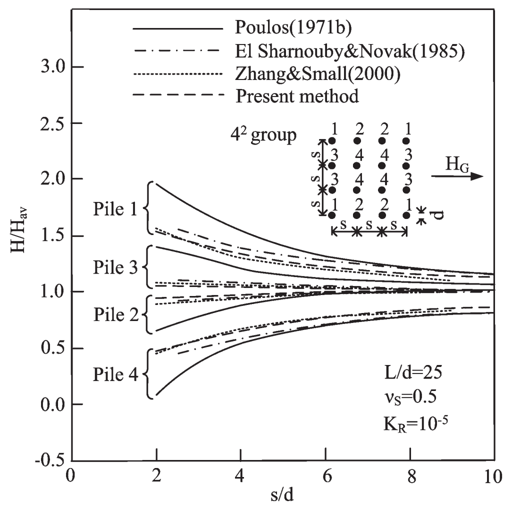

Figure 35. Relationships between the *s*/*d* and the horizontal load distribution in a fixed-head pile group.

5. CONCLUSIONS

The following conclusions can be drawn from the present investigation:

1. For vertically loaded piles, without using the conventional form of the load transfer parameter adopting the maximum radius of the influence of the pile proposed by Randolph and Wroth, the load transfer parameter of a single pile in nonhomogeneous soils is derived from the settlement influence factor concerned with Mindlin's solution for an elastic continuum analysis. For laterally loaded piles, the relationship between the horizontal displacement and stress is produced through the displacement influence factor obtained from Mindlin's solution.
2. The modulus of subgrade reaction along the pile in nonhomogeneous soils is expressed by using displacement influence factors related to Mindlin's solutions for an elastic continuum analysis for vertically and laterally loaded piles to combine the elastic continuum approach with the subgrade reaction approach.
3. The relationship between settlement and vertical load for a single pile subjected to vertical load in nonhomogeneous soils is obtained by using the recurrence equation for each layer.
4. The formulation of interaction factors between pile groups subjected to vertical loads in nonhomogeneous soils is proposed by taking into account Mindlin's equation for the relationship between settlement and shear pressure. Similarly, the formulation of interaction factors between pile groups subjected to horizontal load and moment in nonhomogeneous soils is presented by taking into account Mindlin's equation for the relationship between the horizontal displacement and pressure.
5. By using the first four conclusions, a Winkler model approach is proposed to analyze the behavior of piles subjected to the vertical and lateral loads in nonhomogeneous soils.
6. Instead of using the conventional assumption that a pile is idealized as a thin rectangular vertical strip of width d, length L, and constant flexibility $E_P I_P$, a Winkler model approach is adopted for a circular pile of the diameter d, length L, and constant flexibility $E_P I_P$.
7. The comparison of the results calculated by the present method for single piles and pile groups in nonhomogeneous soils has shown good agreement with those obtained from the more rigorous FEM and BEM.
8. A Winkler model approach proposed has advantages, that is, the analytical solutions for the displacement of a pile subjected to vertical and lateral loads in nonhomogeneous soils are presented and the data are easy to prepare, and does not need creating large meshes as would be required for finite element solutions. It is found that the present procedure gives a good prediction on the behavior of piles in nonhomogeneous soils.

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