Settlement analysis of rectangular piles in nonhomogeneous soil using a Winkler model approach

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SUMMARY

An analytical approach using a Winkler model is investigated to provide analytical solutions of settlement of a rectangular pile subjected to vertical loads in nonhomogeneous soils. For a vertically loaded pile with a rectangular cross section, the settlement influence factor of a normal pile in nonhomogeneous soils is derived from Mindlin’s solution for elastic continuum analysis. For short piles with rectangular and circular cross sections, the modified forms of settlement influence factors of normal piles are produced taking into account the load transfer parameter proposed by Randolph for short circular piles. The modulus of subgrade reaction along a rectangular pile in nonhomogeneous soils is expressed by using the settlement influence factor related to Mindlin’s solution to combine the elastic continuum approach with the subgrade-reaction approach. The relationship between settlement and vertical load for a rectangular pile in nonhomogeneous soils is available in the form of the recurrence equation. The formulation of settlement of soils surrounding a rectangular pile subjected to vertical loads in nonhomogeneous soils is proposed by taking into account Mindlin’s solution and both the equivalent thickness and the equivalent elastic modulus for layers in the equivalent elastic method. The difference of settlement between square and circular piles is insignificant, and the settlement of a rectangular pile decreases as the aspect ratio of the rectangular pile cross section increases. The comparison of results calculated by the present method for a rectangular pile in nonhomogeneous soils has shown good agreement with those obtained from the analytical methods and the finite element method. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: rectangular pile; nonhomogeneous soil; settlement; Winkler model; Mindlin’s solution; stiffness coefficient

1. INTRODUCTION

For designs and analyses of pile foundations, special attention for piles has been recently concentrated on not only the behavior of a pile with a circular cross section but also those of piles with other cross sections such as a rectangular cross section and a rectangular cross section with rounded corners. Piles with the other cross sections have been often used. Barrette piles, which are reinforced concrete piles with rectangular cross sections, have the resistance to horizontal loads and to bending moments better than circular piles of the same cross-sectional area. The contiguous and secant piles, which are modeled as those with a rectangular cross section or a rectangular cross section with rounded corners and are composed of the improved soil or the reinforced concrete, have been used in the prevention of settlement and liquefaction. However, analyses of piles with the other cross sections have used often the method where those are converted to a pile with a circular cross section of an equivalent area.

Parametric solutions of circular piles have been produced for various practical cases, and most of them are related to homogeneous soils. For example, the integral equation method or the boundary...
element method (BEM) given by Butterfield and Banerjee [1] has been used to provide the numerical computer-based solutions for circular piles in homogeneous soils. For nonhomogeneous soils, Banerjee and Davies [2, 3], Banerjee [4], Poulos [5, 6], Poulos and Davis [7], Chow [8–10], Lee [11], Southcott and Small [12], Ta and Small [13–15], Zhang and Small [16], Small and Zhang [17], and Kitiyodom and Matsumoto [18] presented the solutions of circular pile foundations, using the numerical methods such as the finite element method (FEM) and BEM and the simplified analytical approaches.

It is assumed in a rigorous analysis that the pile is a three-dimensional continuum embedded in the extended half-space soil that is a three-dimensional elastic medium. Because the problem of a vertically loaded pile is axisymmetric, the pile and the soil each take two displacement components in the radial and vertical directions for the rigorous analysis. The rigorous analytical approach ensures the boundary, continuity, and compatibility conditions of the displacement, stress, force, rotation, and moment for the pile and half-space. The fundamental work for the rigorous analytical approach is done by Muki and Sternberg [19, 20], who investigated the diffusion and transfer of the axial load from a long cylindrical elastic bar into the surrounding elastic medium. Luk and Keer [21] presented a rigorous analytical formulation for the problem of a rigid cylindrical inclusion partially embedded in an isotropic elastic half-space in the case of the axial loading. Selvadurai and Rajapakse [22] demonstrated a rigorous analytical method related to the axial loading of a rigid cylindrical inclusion embedded in an isotropic elastic half-space. Rajapakse and Shah [23–25] presented a rigorous analytical method to solve an elastic circular bar embedded in an elastic half-space subjected to the longitudinal load. Rajapakse [26] investigated rigorously the response of an axially loaded elastic pile in a Gibson soil using a variational method.

It is assumed in a less rigorous analysis for a vertically loaded pile that the pile and the soil each have one displacement component in the vertical direction. The less rigorous analytical approach has several limitations such as the absence of the displacement, stress, force, rotation, and moment and the neglect of the boundary, continuity, and compatibility conditions. Various improvements to the work of the less rigorous approach have been reported by many researchers. Poulos [5, 6], Poulos and Davis [7, 27], and Mattes and Poulos [28] employed a finite difference method to analyze the behavior of vertically loaded single circular piles and pile groups. Randolph and Wroth [29] presented approximate closed-form linear elastic solutions for settlement of a circular pile in homogeneous and nonhomogeneous soils. Randolph and Wroth [30] developed the method of using the closed-form solution for a vertically loaded single circular pile to produce the solution for vertically loaded circular pile groups. Poulos and Davis [7] presented both an analytical solution for a single floating circular pile and a solution based on an iterative method for a single end-bearing circular pile in nonhomogeneous soils by introducing an average elastic modulus for the soil. Guo and Randolph [31] and Guo [32] investigated analytically the response of vertically loaded circular piles in elastic-plastic, nonhomogeneous soils.

For vertically loaded circular piles, developing the Winkler model of soil reaction for the interaction between circular piles and soils in layered soils, Mylonakis and Gazetas [33] proposed analytical expressions for settlement and interaction factors. Mylonakis [34] investigated analytically the modulus of subgrade reaction based on the Winkler model for axially loaded circular piles. Using Mindlin’s solution for vertical loads, Yamaha [35] proposed the stiffness coefficients of surface foundation defined as the ratio of the vertical load to the displacement of the rigid rectangular base on the surface of the semi-infinite solid. For a pile with a circular cross section in nonhomogeneous soils, Hirai [36] presented an analytical approach based on the elastic continuum analysis using a Winkler model that is assumed through the modulus of subgrade reaction for the relationship between the shear stress and settlement for the vertical loading.

When a circular pile is used originally in an analysis, the circular pile is often modeled as a square pile by replacing the circular pile with the square pile of equivalent cross-sectional area in the numerical method such as FEM to reduce the number of elements, for example, Kwon and Elnashai [37]. On the other hand, analytical methods developed for a circular pile have been applied to a rectangular pile by replacing the rectangular pile with the circular pile of the same cross-sectional area. Small and Zhang [17] pointed out that the replacement of a circular pile with a square pile of the same cross-sectional area makes them equivalent for the vertical loading, but because the second moment of area is larger for the square pile, the bending stiffness is 4.7% higher. To investigate the
behavior of pile groups embedded in Gibson soil using BEM, Banerjee and Davies [2] adopted a method where a circular pile is replaced with a square pile of the same surface area. Basu et al. [38] and Seo et al. [39] presented a method of the settlement analysis that is applied to axially loaded piles with a rectangular cross section installed in multilayered soil deposits, and the analysis follows from the solution of the differential equations governing the displacements of pile–soil system obtained using variational principles. At the present time, the analytical solutions for rectangular or square piles are not available in the literature besides Basu et al. [38] and Seo et al. [39]. This implies that the analytical procedure for the rectangular pile may be much more complicated than that for circular piles. The method proposed by Basu et al. [38] and Seo et al. [39] incorporates the effect of Poisson’s ratio into the expression for the modified shear modulus. However, it may be necessary that the effect of Poisson’s ratio and that of the original shear modulus on the settlement are investigated separately to evaluate each effect on the settlement directly.

In the following presentation, an investigation for a pile with a rectangular cross section in nonhomogeneous soils is made to propose approximate analytical solutions of the settlement for vertical loads. First, for a vertically loaded pile with a rectangular cross section, the settlement influence factor of normal piles (with the length-diameter ratio of 10 or more) in nonhomogeneous soils is derived from Mindlin’s solution for elastic continuum analysis. For short piles with rectangular and circular cross sections, the modified forms of the settlement influence factors of normal piles are produced taking into account the load transfer parameter proposed by Randolph [40] for short circular piles. Second, to combine the elastic continuum approach with the subgrade-reaction approach, the modulus of subgrade reaction along the rectangular pile in nonhomogeneous soils for vertical loads is expressed by the settlement influence factor related to Mindlin’s solution in elastic continuum analysis. Third, the relationship between settlement and vertical load for a rectangular pile subjected to the vertical load in nonhomogeneous soils is obtained using the recurrence equation for each layer. Fourth, the formulation of settlement of soils surrounding a rectangular pile subjected to vertical loads in nonhomogeneous soils is proposed by taking into account Mindlin’s solution in a homogeneous soil and both the equivalent thickness and the equivalent elastic modulus for layers in the equivalent elastic method [41]. Subsequently, a Winkler model approach of rectangular piles is proposed to analyze the settlement of piles with a rectangular cross section subjected to the vertical loads in nonhomogeneous soils. It is assumed conventionally that a rectangular pile is idealized as a circular pile with the same cross-sectional area in analytical methods. On the other hand, a circular pile is often assumed to be a square pile with the equivalent area in numerical methods such as FEM. However, it seems that the investigation for the validity of these assumptions has been hardly made clearly. Therefore, the continuum-based approach used for circular piles proposed by Hirai [36] will be developed for the analysis of rectangular piles. The comparison of the results calculated by the present method for rectangular and circular piles in nonhomogeneous soils is performed with those obtained from the analytical methods and FEM.

2. FORMULATION OF SETTLEMENT OF A RECTANGULAR PILE SUBJECTED TO VERTICAL LOAD

To obtain a solution for the values of shear stress along a pile and settlement of the pile, it is necessary to give expressions for settlement of the pile and soil at each element in terms of the unknown shear stresses on the pile. Figure 1 shows a rectangular or circular pile discretized into several segments of 1~mb~1 in nonhomogeneous soils, with mb denoting the mbth soil layer beneath the base of the pile subjected to a vertical load. A Cartesian coordinate system (x, y, z) is used for the rectangular pile. As shown in Figure 1, the present procedure uses the elastic moduli, that is, Young’s modulus E_m, Poisson’s ratio ν_m, and thickness H_m for the mth layer in the n layers of nonhomogeneous soils; L is the length of a pile; B_x and B_y are the dimensions of the rectangular pile cross section in the x and y directions, respectively; d is the diameter of the circular pile shaft; P_{pm}, P_{sm}, and P_{bm} are the axial force, the shear force, and the base force of the mth element, respectively; and w_{pm} and w_{bm} are settlements at the head and base of the mth element, respectively.

Figure 2 shows discretized elements for a rectangular pile subjected to a vertical load on multilayered soil. At a depth coordinate c of the point where the shear stress p_{sm} is applied to the
element \( m \) in the pile shaft, the distance \( r \) is taken between the plane coordinates \((x, y)\) of the point where the shear stress is applied to the element \( m \) and the origin of the coordinate system, and the distance \( r' \) is taken between the plane coordinates \((x_0, y_0)\) at a depth coordinate \( h \) of the point where the settlement is evaluated in the soil medium. For the base of a pile, the distance \( r \) is taken between the coordinates \((x, y)\) of the point where the vertical pressure \( p_{B}\) \((\frac{m}{C_1})\) is applied on the pile base and the origin of the coordinate system, the distance \( r' \) is taken between the coordinates \((x, y)\) and the coordinates \((x_0, y_0)\), and the distance \( R \) is taken between the coordinates on the perimeter of the rectangular pile cross section through the coordinates \((x, y)\) and the origin of the coordinate system. For the coordinate system from the pile base, \( c' \) is a depth coordinate from the bottom of the point to which the vertical load is applied, and \( h' \) is a depth coordinate from the bottom of the point at which the settlement is evaluated.

For multilayered soil having the vertical nonhomogeneity with some variation of stiffness with depth, by making reference to equations given by Poulos and Davis [7], the settlement at a depth of the soil that is adjacent to the pile subjected to the shear stress \( p_S \) along the pile may be written in the Cartesian coordinates \((x, y, z)\) as follows:

\[
w = w(x_0, y_0, h) = \int_0^{L} \left\{ \int_{-B_z/2}^{B_z/2} p f(x_0, y_0, h, B_z/2, y, c, v(c)) \cdot dy \right. \\
+ \int_{-B_z/2}^{B_z/2} p f(x_0, y_0, h, -B_z/2, y, c, v(c)) \cdot dx \right\} \frac{p_S(c)}{E(c)} dc \\
+ \int_{-B_z/2}^{B_z/2} p f(x_0, y_0, h, x, B_z/2, c, v(c)) \cdot dy \right\} p_S(c) \cdot dc \\
+ \int_{-B_z/2}^{B_z/2} p f(x_0, y_0, h, x, -B_z/2, c, v(c)) \cdot dx \right\} \frac{p_S(c)}{E(c)} dc
\]  

(1)
where \( w \) is the settlement of the soil; \( \rho I \) is the influence factor for vertical displacement as a result of a vertical point load, which is represented by Mindlin’s solution; and \( E(c) \) and \( \nu(c) \), which depend on the depth, are Young’s modulus and Poisson’s ratio of the soil, respectively. Referring to Figure 2, the solution of an integral form given by Eqn (1) for the settlement evaluated in the \( i \)th layer in a multilayered soil has a summation of settlement which is produced by the shear stress \( p_{sm} \) applied to the \( m \)th layer in the multilayered soil. Also, Eqn (1) is an analytical solution of a multilayered medium, which is represented by the summation of each Mindlin’s solution for a local homogeneous continuum.

Let us consider a simplified form of the relationship between the settlement and the shear stress obtained by performing the transformation from Eqn (1). It may be noticed in Eqn (1) that the integrand is a function of the depth coordinate \( c \) over the length of the pile, and the function \( \rho I \) specified later possesses a singularity at the local field point \( c = h \) where the settlement is evaluated. In this case, applying the first mean value theorem for integration (Gradshteyn and Ryzhik [42]) to Eqn (1) and taking into account the singularity of the function of \( \rho I \) at the depth \( c = h \), we may assume approximately as follows:

\[
w = w(x_0, y_0, h) = I_v(x_0, y_0, h) \cdot \frac{P_S(h)}{E(h)} \tag{2}
\]

where \( p_S(h), E(h), \) and \( \nu(h) \) vary with depth \( h \), and \( I_v(x_0, y_0, h) \) is the settlement influence factor. If it is assumed for convenience that Poisson’s ratio of the soil at the depth \( h \) where the settlement is calculated is predominant as well as Young’s modulus of the soil and the shear stress at the depth \( h \), the settlement influence factor \( I_v(x_0, y_0, h) \) can be written as follows:
\[ I_v = I_v(x_0, y_0, h) = I_v(x_0, y_0, h, v(h)) \]

\[ = \int_0^{B_y/2} \int_{-B_x/2}^{B_x/2} pI(x_0, y_0, h, B_x/2, y, c, v(h)) \, dy \, dx \]

\[ + \int_{-B_x/2}^{B_x/2} \int_{-B_y/2}^{B_y/2} pI(x_0, y_0, h, x, B_y/2, c, v(h)) \, dx \, dy \]

\[ + \int_{-B_y/2}^{B_y/2} \int_{-B_x/2}^{B_x/2} pI(x_0, y_0, h, -B_x/2, y, c, v(h)) \, dy \, dx \]

\[ + \int_{-B_x/2}^{B_x/2} \int_{-B_y/2}^{B_y/2} pI(x_0, y_0, h, x, -B_y/2, c, v(h)) \, dx \, dy \] \text{ (3)}

where \( pI \) is given by Mindlin’s solution as follows:

\[
pI = \frac{1 + \nu}{8\pi(1-\nu)} \left\{ \frac{z_1^2}{D_1^3} + \frac{3 - 4\nu}{D_1} + \frac{5 - 12\nu + 8\nu^2}{D_2} \right. \\
\left. + \frac{(3 - 4\nu)z_2^2 - 2cz_2 + 2c^2}{D_2^3} + \frac{6cz_2^2(z_2 - c)}{D_2^5} \right\} 
\]

\[ z_1 = h - c \]

\[ z_2 = h + c \]

\[ r^2 = (x_0 - x)^2 + (y_0 - y)^2 \]

\[ D_1^2 = r^2 + z_1^2 \]

\[ D_2^2 = r^2 + z_2^2 \] \text{ (4)}

For Eqn (2), which is a simplified form obtained from Eqn (1), the settlement influence factor \( I_v \), given by Eqn (3) for the point where the settlement is evaluated in the \( i \)th layer in a multilayered soil takes the summation of Mindlin’s solution that represents the property of the local homogeneous \( m \)th soil layer that the shear stress \( p_{sm} \) is applied to. Hence, an analytical solution for a multilayered soil is specified as Eqn (1), and a simplified form of the analytical solution for the multilayered soil is expressed as Eqn (2), which provides the Winkler model relationship between the settlement and the shear stress.

Assuming that the settlement influence factor given by Eqn (3) is a function of the depth \( h \) only, an averaged value of the settlement influence factor over the rectangular cross section at a depth \( h \) can be provided. In this case, because the complexity of the triple integral obtained from Eqn (3) yields using the integral form to obtain an average value of the settlement influence factor along the periphery of a pile, a simplified form of the settlement influence factor for the rectangular pile is presented in the following. A settlement influence factor \( I_v = I_v(h, v(h)) \) representative of settlement influence factors \( I_v(x_0, y_0, h, v(h)) \) at the coordinates \( (x_0, y_0) \) on the plane at the depth \( h \) for a pile with a rectangular cross section is taken as an average of three values that are the settlement influence factor \( I_v(x_0, y_0, h, v(h)) \) on the corner point and those on the middle points in the dimensions \( B_x \) and \( B_y \) of a pile with a rectangular cross section. Thus, the settlement influence factor \( I_v = I_v(h, v(h)) \) of the rectangular pile at the depth \( h \) is written as follows:

\[ I_v = I_v(h, v(h)) = \left\{ I_v(B_y/2, B_y/2, h, v(h)) + I_v(B_x/2, 0, h, v(h)) + I_v(0, B_y/2, h, v(h)) \right\} / 3 \] \text{ (5)}

From Eqn (2), the settlement \( w_m \) of the soil adjacent to the \( m \)th element of a rectangular pile subjected to the shear stress \( p_{sm} \) \( (m = 1 \sim mb - 1) \) along the pile may be expressed as follows:

\[ w_m = I_{vrm} \frac{p_{sm}}{E_m} \left( m = 1 \sim mb - 1 \right) \] \text{ (6)}

where \( I_{vrm} \) is the settlement influence factor related to the soil adjacent to the \( m \)th element of the rectangular pile. Eqs (2) and (6) imply that an integral form of the settlement influence factor \( I_v \) is calculated for a given local field point at the depth \( h \), taking into account the characteristics that the integrand is a function of the coordinates at the depth \( c \) and Poisson’s ratio over the length of the pile and possesses a singularity at the local field point \( c = h \) and then applying this to calculation of
the local settlement at the depth \( h \), taking the local values at the depth \( h \) of the shear stress, Young’s modulus and Poisson’s ratio. For a pile with the circular cross section subjected to a vertical load, an analytical approach similar to the formulation presented in Eqns (1–6) was proposed by Hirai [36].

Randolph and Wroth [29], Randolph [40], and Horikoshi and Randolph [43] pointed out that the load transfer parameter proposed by Randolph and Wroth [29] is accurate for normal circular piles (with the length–diameter ratio \( L/d \) of 10 or more) but is not suitable for short circular piles having the small length–diameter ratio \( L/d \). Although Eqn (1) is represented following the method proposed by Poulos and Davis [7], Eqn (6) obtained from Eqn (1) is the form similar to the relationship between the settlement and the shear stress proposed by Randolph and Wroth [29]. Thus, once Eqn (6) is obtained, the analytical procedure following it is analogous to that given by Randolph and Wroth [29]. Adjusting the load transfer parameter of normal circular piles empirically, Randolph [40] presented a form of the load transfer parameter of short circular piles. Taking into account that the load transfer parameter of short circular piles proposed by Randolph [40] has an addition of the constant term, it may be assumed that the settlement influence factor, \( I_{vcM} \), of short piles with a circular cross section may be written in the modified form as follows:

\[
I_{vcM} = (1 + \nu_s) \ln \left\{ \frac{h_{RB}}{e^{\frac{\nu_s}{1 + \nu_s} + 5}} \right\} 
\]

where \( I_{vc} \) is the settlement influence factor proposed originally for normal piles with a circular cross section [36], and the settlement influence factors \( I_{vc} \) and \( I_{vcM} \) have no units. Because Eqn (7) is an interpolation function to transform from the settlement influence factor of the normal circular pile to that of the short circular one, the relevant length–diameter ratio \( L/d \) of a short pile is assumed to be up to the value of 10.

Considering a differential equation of settlement for a rectangular pile obtained from equilibrium of external forces and taking into account both a circular pile whose cross-sectional area is equal to that of short circular piles, the settlement in a rectangular cross section, and the settlement in a circular cross section may be represented in the modified form as follows:

\[
I_{vrM} = 2(B_x + B_y)(1 + \nu_s)/\pi \ln \left\{ \frac{h_{RM}}{e^{\frac{\nu_s}{1 + \nu_s} + 5}} \right\} 
\]

where \( I_{vr} \) is the settlement influence factor presented in Eqn (5) proposed originally for normal piles with a rectangular cross section, and the settlement influence factors \( I_{vr} \) and \( I_{vrM} \) have the units of length. Because Eqn (8) is an interpolation function to transform from the settlement influence factor of the normal rectangular pile to that of the short rectangular one, the relevant length–equivalent diameter ratio \( L/d \) of a short pile is assumed to be up to the value of 10. The equivalent diameter \( d_{eq} \) is equal to the diameter \( d \) for a circular pile and \( 2(B_xB_y)^{1/2} \) for a rectangular pile whose cross-sectional area is equal to that of a circular pile. Hence, it follows from Eqns (7) and (8) that the settlement influence factors, \( I_{vcM} \) and \( I_{vrM} \), of short piles approach those, \( I_{vc} \) and \( I_{vr} \), of normal piles, respectively, as the length–equivalent diameter ratio \( L/d \) and the length–equivalent diameter ratio \( L/d \) approach the upper value of 10.

For the soil layers of finite depth, the settlement may be approximately obtained by using the Steinbrenner [44] approximation that the compression of a finite depth deposit. Thus, the settlement \( w \) at a depth \( h \) in a layer of depth \( h_{RB} \) is written as

\[
w = w(x_0, y_0, h, v(h)) = I_{RB}(x_0, y_0, h, v(h)) \frac{P_s(h)}{E(h)} 
\]

where \( I_{RB}(x_0, y_0, h, v(h)) \) is the settlement influence factor expressed as \( I_{RB}(x_0, y_0, h, v(h)) = I_s(x_0, y_0, h, v(h)) \), of short piles, and \( I_{RB}(x_0, y_0, h, v(h)) \), of short piles.

For the Winkler soil model of subgrade-reaction analysis, the relationship between the shear stress \( p_s \) and deflection \( w \) at the depth \( h \) in nonhomogeneous soils subjected to vertical loads is assumed to be related as follows:

\[
p_s = k_s(h)w 
\]

where \( k_s(h) \) is the modulus of subgrade reaction and varies with the depth \( h \). The modulus of subgrade reaction \( k_s(h) \) in Eqn (10) along the pile in nonhomogeneous soils is derived from the relationship
between settlement and shear stress in an elastic continuum represented in Eqn (2) to establish the relationship between the elastic continuum approach and the subgrade-reaction approach for piles subjected to vertical loads, as follows:

\[
k_v(h) = \frac{E(h)}{I_v(h, v(h))}
\]

(11)

Let us consider the settlement of a single pile and the interaction between piles taking into account the axial, the shear, and the base forces. As shown in Figures 1 and 2, in the case where the axial force \( P_P \) on the top of a pile in the first soil layer is applied, the \( m \)th element of the pile is loaded with the axial force \( P_{Pm} \) (\( m = 1 \sim mb - 1 \)) on the head, the vertical shear force \( P_{Sm} \) around the periphery, and the base force \( P_{Bm} \). These forces satisfy the vertical equilibrium of the \( m \)th pile element, that is, \( P_{Pm} = P_{Sm} + P_{Bm} \). It is assumed that the settlement of the pile element is identical with that of the soil element adjacent to the pile element.

For the case where the head of the pile \( k \) is subjected to external load \( P_{Plk} \), the settlement \( w_{Pl} \) of the head of the pile \( i \) is represented as

\[
w_{Pl} = \sum_{k=1}^{N} F_{Pik} \cdot P_{Plk}
\]

(12)

where \( N \) is the number of piles, and for the settlement of a single pile,

\[
F_{Pik} = F_1 \quad (i = k)
\]

(13)

and for the interaction between piles,

\[
F_{Pik} = F_1' \quad (i \neq k)
\]

(14)

Using the formulation presented by Randolph and Wroth [29] and Hirai [36] in the case of a circular pile and taking into account the vertical equilibrium between the axial, the shear, and the base forces, the parameter \( F_1 \) in Eqn (13) is obtained by the following recurrence equation:

\[
F_m = \frac{F_{m+1} + FA_m}{FB_m F_{m+1} + 1} \quad (m = 1 \sim mb - 1)
\]

(15)

where

\[
FA_m = \frac{1}{\mu_m A_P E_p} \tanh(\mu_m H_m)
\]

\[
FB_m = \mu_m A_P E_p \tanh(\mu_m H_m)
\]

(16)

\[
\mu_m = \left\{ \frac{2(B_x + B_y) E_m}{A_p E_p I_{prm}} \right\}^{1/2}
\]

where \( A_P \) is the area of the rectangular pile cross section and \( E_p \) is the elastic modulus of the pile.

The initial value \( F_{mb} \) in Eqn (15) is obtained from the relationship between settlement and load for the pile base on multilayered soils. Hirai [41] proposed the relationship between settlement and load for the circular pile base on multilayered soils on the basis of both the equivalent thickness and the equivalent elastic modulus for layers in the equivalent elastic method. Considering that the pile base with a rectangular cross section is represented as a rigid punch acting on the surface of soils ignoring the pile shaft and surrounding soil depth and taking into account Mindlin’s solution in a homogeneous soil and the equivalent elastic method proposed by Hirai [41], the relationship between the settlement \( w_{B(mb-1)} \) on the pile base and the base load \( P_{B(mb-1)} \) and that between the vertical pressure \( p_{B(mb-1)} \) and the base load \( P_{B(mb-1)} \) for the rigid base of a rectangular pile are obtained as follows:

\[
w_{B(mb-1)} = F_{mb} P_{B(mb-1)}
\]

\[
P_{B(mb-1)} = \frac{P_{B(mb-1)}}{\left( R^2 - r^2 \right)^{1/2} A_1}
\]

(17)
where

\[ F_{mb} = \frac{1}{A_1} \left\{ I_p \left( H_{mb} \cdot v_n \right) \right\} + \sum_{m=mb+1}^{n} \frac{I_p \left( \sum_{j=mb}^{m} H_{je} \cdot v_n \right)}{E_m} \]

\[ I_p \left( h', v_n \right) = K \left( 0, v_n \right) - K \left( h', v_n \right) \]

\[ K \left( h', v_n \right) = 4 \left\{ \int_0^{b_1/2} \frac{\partial K_1}{\partial y} A_2 dy \right\} + \int_0^{b_1/2} A_2 dy \]

\[ A_1 = 2 \left\{ B_1 \ln \left( \frac{B_y}{B_x} \right) + \left( 1 + \frac{B_y^2}{B_x^2} \right)^{1/2} \right\} \]

\[ A_2 = \frac{p^2}{(R^2 - r^2)^{1/2}} \]

where \( x_0 = y_0 = c' = 0 \) and \( H_{je}' \) is the equivalent thickness [41] of the \( j \)th soil layer in the equivalent elastic method. In the case of a single layer beneath the pile base, the settlement \( w_{B(mb-1)} \) on the pile base in Eqn (17) is reduced to the equation given by Yamahara [35] in the following form:

\[ w_{B(mb-1)} = \frac{\pi (1 - v_{mb}^2)}{E_{mb}A_1} P_{B(mb-1)} \]  

(19)

Let us consider the formulation of the parameter \( F_i \) represented by Eqn (14) regarding the interaction between piles. For the case where the head of the pile \( k \) is subjected to the external load \( P_{P1k} \) and the shear stress on the \( m \)th segment of the shaft of the pile \( k \) produced by the load \( P_{P1k} \) is \( P_{Smk} \), the settlement \( w_{Pik} \) of the pile \( i \) may be given as follows:

\[ w_{Pik} = w(x_0, y_0, h) \]

\[ = \frac{\pi}{2} \left\{ \int_0^{b_1/2} I_p \left( x_0, y_0, h, B_x/2, y, c, v(c) \right) \cdot dy \right\} \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, x, B_x/2, c, v(c) \right) \cdot dx \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, -B_x/2, y, c, v(c) \right) \cdot dy \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, -B_x/2, c, v(c) \right) \cdot dx \}

\[ \frac{P_{ik}(c)}{E(c)} dc \]

\[ = F_{Pik} \cdot P_{P1k} \]

where

\[ F_{Pik} = \frac{1}{2(B_x + B_y)} \sum_{m=1}^{mb-1} I_{link} \cdot F_{P_{sik}} \]

\[ I_{link} = I_{link} \left( x_0, y_0, h \right) \]

\[ = \int_{TH_{mk}}^{b_1/2} I_p \left( x_0, y_0, h, B_x/2, y, c, v(c) \right) \cdot dy \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, x, B_x/2, c, v(c) \right) \cdot dx \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, -B_x/2, y, c, v(c) \right) \cdot dy \]

\[ + \int_{-b_1/2}^{b_1/2} I_p \left( x_0, y_0, h, -B_x/2, c, v(c) \right) \cdot dx \}

\[ \frac{P_{sik}(c)}{E(c)} dc \]

\[ F_{P_{sik}} = P_{Smk} / P_{P1k} \]

where \( P_{Smk} \) is the shear force on the \( m \)th segment of the shaft of the pile \( k \) subjected to the load \( P_{P1k} \) and \( TH_{mk} = \sum_{i=1}^{N} H_i \). For piles of \( k = 1 \rightarrow N \), the settlement \( w_{P/ik} \) of the pile \( i \) is written using Eqn (20) as follows:
\[ w_{Pi}^{' } = \sum_{k=1}^{N} w_{Pik}^{' } = \sum_{k=1}^{N} F_{Pik}^{' } \cdot P_{P1k} \] (22)

For the case where the head of the pile \( k \) is subjected to external load \( P_{P1k} \) and the stress acting on the rigid pile base is \( p_{B(m-b-1)k} \), the settlement \( w_{P1k}'' \) of the pile \( i \) is expressed as follows:

\[ w_{P1k}'' = F_{P1k}'' \cdot P_{P1k} \] (23)

where

\[ F_{P1k}'' = \frac{1}{A_1E_{beq}} \cdot I_{iBk} \cdot F_{P_{B(m-b-1)k}P_{P1k}} \]

\[ I_{iBk} = I_{iBk}(x_0, y_0, h, c) \]

\[ = \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy + \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy - \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy \]

\[ + \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy + \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy - \int_{0}^{B_y/2} \int_{0}^{B_x/2} A_2 dx dy \] (24)

\[ F_{P_{B(m-b-1)k}P_{P1k}} = P_{B(m-b-1)k} / P_{P1k} \]

where \( E_{beq} \) is the equivalent elastic modulus \([41]\) for layers beneath the pile base in the equivalent elastic method and \( P_{B(m-b-1)k} \) is the vertical force acting on the rigid base of the pile subject to the load \( P_{P1k} \). For piles of \( k = 1 \sim N \), the settlement \( w_{P1i}'' \) of the pile \( i \) is expressed using Eqn (23) as follows:

\[ w_{P1i}'' = \sum_{k=1}^{N} w_{P1k}'' = \sum_{k=1}^{N} F_{P1k}'' \cdot P_{P1k} \] (25)

Thus, in the case where the pile \( k \) \((k = 1 \sim N)\) is subjected to the load \( P_{P1k} \), the settlement \( w_{Pi} \) of the pile \( i \) is written by Eqns (22) and (25) as follows:

\[ w_{Pi} = w_{Pi}^{'} + w_{Pi}'' \]

\[ = \sum_{k=1}^{N} \left( F_{Pik}^{'} + F_{Pik}'' \right) \cdot P_{P1k} \] (26)

where

\[ F_{P1k} = F_{P1k}^{'} + F_{P1k}'' \] (27)

Therefore, it is found from Eqns (14) and (27) in the case of the depth \( h = 0 \) that

\[ F_{1}^{'} = F_{P1k}^{'} + F_{P1k}'' \quad (i \neq k) \] (28)

Furthermore, Eqns (12–28) can be used to obtain the settlement of the soil surrounding the pile \( k \) subjected to vertical loads by replacing the coordinates of the pile \( i \) with those of the soil.
3. NUMERICAL RESULTS

Let us consider the settlement of a single rectangular pile subjected to a vertical load on the surface of a nonhomogeneous soil. It is of interest to investigate the difference between the stiffness coefficient of a pile with a rectangular cross section and that of a pile with a circular cross section. For the stiffness coefficient [45–47] generally used in elastodynamics, the vertical stiffness coefficient of a soil adjacent to a pile is written as \( K_v = \frac{P_s}{w}, \) where \( P_s = \pi dp_S \) and \( P_s = 2(B_x + B_y)p_{S} \) are the vertical shear force per unit length of a circular pile and that of a rectangular pile, respectively, and \( w \) is the settlement of a soil adjacent to the pile. The load transfer parameter \( \zeta \) of a pile with a circular cross section given by Randolph and Wroth [29] can be determined using the conventional form

\[
\zeta = \ln(2r_m/d) \quad \text{of adopting the maximum radius of influence of a pile, } r_m.
\]

For a nonhomogeneous soil, the maximum radius of influence of a pile is defined as \( r_m = 2.5\rho(1-\nu_S)L, \) where \( \rho \) is a nonhomogeneity factor that is the ratio of the shear modulus \( G(h) \) of the soil at the pile middepth to that at the base, that is, \( \rho = G(L/2)/G(L), \) and \( \nu_S \) is Poisson’s ratio of the soil. Thus, the vertical stiffness coefficient of a pile with a circular cross section obtained from Randolph and Wroth [29] is represented as

\[
K_v = \pi E_S I_v/(1 + \nu_S)\zeta
\]

where \( I_v \) is Young’s modulus of the soil. Hirai [36] proposed the vertical stiffness coefficient of a pile with a circular cross section in the following form:

\[
K_v = \pi E_S I_v/(1 + \nu_S)\zeta
\]

In the following presentation, the results predicted for piles with square and rectangular cross sections are obtained using settlement influence factors expressed by Eqns (5) and (8), and the results predicted for a pile with a circular cross section are obtained from analytical solutions that Hirai [36] proposed using a Winkler model approach for the vertically loaded pile in a nonhomogeneous soil and the settlement influence factor expressed by Eqn (7).

Figures 3 and 4 show relationships between the depth ratio \( z/L \) and the vertical stiffness coefficients \( K_v, K_{vc}, \) and \( K_{vr} \) for homogeneous soils having \( \nu_S = 0.0 \) and \( \nu_S = 0.5, \) respectively. It is seen that Poisson’s ratio has a significant influence on the distribution of the vertical stiffness coefficients. The vertical stiffness coefficients of a pile with a square cross section \( K_{vr} \) is slightly larger than that of a pile with a circular cross section \( K_v. \) The difference between the vertical stiffness coefficients \( K_v \) and \( K_{vc} \) is that \( K_v \) takes a constant value dependent on the slenderness ratio of a pile \( L/d_c \) and Poisson’s ratio of the soil, whereas \( K_{vc} \) depends on the depth ratio \( z/L, \) the length–diameter ratio \( L/d_c, \) and Poisson’s ratio of the soil. Further, it is found that the value at the middepth of the vertical stiffness coefficient \( K_{vc} \) gives good approximation of \( K_v. \)

Figures 5 and 6 show relationships between the depth ratio \( z/L \) and the vertical stiffness coefficient \( K_v \) of the rectangular pile for a homogeneous soil having \( \nu_S = 0.0 \) and \( \nu_S = 0.5, \) respectively. As the aspect ratio \( B_x/B_y \) increases and the length–diameter ratio \( L/d_c \) decreases, the vertical stiffness coefficient of a pile with a rectangular cross section increases. As pointed out by Seo et al. [39], the additional stiffness with the increase of the aspect ratio is considered to be produced by the larger perimeter of the square or rectangular pile for the same cross-sectional area.

![Graph showing relationships between z/L and the vertical stiffness coefficient for circular and square piles.](image-url)
Figure 7 shows relationships between the length–diameter ratio \( L/d \), the diameter–length ratio \( d/L \), and the displacement in influence factor \( I_w \) of a pile with a circular cross section for two Poisson’s ratios \( \nu_S = 0 \) and \( \nu_S = 0.5 \). Results obtained from the settlement in influence factor presented in Eqn (7) are compared with those obtained from Poulos and Davis [27] and Randolph [40] and show good agreement.

To compare the results from the present approach with numerical and analytical solutions available in the literature, the normalized pile head stiffness coefficient \( F_v \) is defined as the ratio of the pile head stiffness coefficient \( K_{vp} \) of a pile with some type of cross section to the stiffness coefficient \( K_{v0} \) of the surface foundation with the rigid circular base. The pile head stiffness coefficient \( K_{vp} \) is defined as the ratio of the vertical load to the displacement on the pile head, that is, \( K_{vp} = P/P_{wp} \). The stiffness
Coefficient $K_{v0}$ of the surface foundation with the rigid circular base is defined as the ratio of the vertical load to the displacement of the rigid circular base with the equivalent diameter $d_e$ on the surface of the semi-infinite solid and is written as $K_{v0} = E_Sd_e/(1 - \nu_S^2)$.

Figure 8 shows relationships between the length–diameter ratio $L/d$ and the normalized pile head stiffness coefficient $F_v$ of the circular pile in the case of Poisson’s ratio $\nu_S = 0.25$. As the length–diameter ratio $L/d$ and the stiffness ratio $E_p/E_S$ increase, the normalized pile head stiffness coefficient $F_v$ increases. Results obtained from the settlement influence factor presented in Eqn (7) are compared with those obtained from Rajapakse and Shah [23, 25], Randolph and Wroth [29], Randolph [40], and Apsel and Luco [48] and show good agreement.

Figure 9 shows relationships between the length–diameter ratio $L/d_e$ and the normalized pile head stiffness coefficient $F_v$ of a pile with a circular cross section in the case of Poisson’s ratios $\nu_S = 0.0$ and $\nu_S = 0.5$. As the length–diameter ratio $L/d_e$ and the stiffness ratio $E_p/E_S$ increase, the normalized
pile head stiffness coefficient increases and that for Poisson’s ratio $\nu_S = 0.0$ is larger than that for $\nu_S = 0.5$. There is a small difference between the results obtained from Seo et al. [39] and those given by the present method.

For a short pile with a square cross section, let us investigate in detail the difference between settlements obtained from the original and modified forms of the settlement influence factor presented in Eqns (5) and (8), respectively. Figure 10 shows relationships between the length–diameter ratio $L/d_e$ and the normalized pile head stiffness coefficient $F_v$ of a pile with a square cross section for two Poisson’s ratios $\nu_S = 0.0$ and $\nu_S = 0.5$ and two stiffness ratios $E_p/E_s = 500$ and $E_p/E_s = 10^5$. As the length–diameter ratio $L/d_e$ increases, the normalized pile head stiffness coefficient $F_v$ obtained from the modified form expressed by Eqn (8) approaches that obtained from the original form presented in Eqn (5).

To investigate the settlement for the overall range of the length–diameter ratio $L/d_e$ for a pile with a square cross section, Figure 11 shows relationships between the length–diameter ratio $L/d_e$ and the...
normalized pile head stiffness coefficient $F_v$ for a pile with a square cross section. As the length–diameter ratio $L/d_0$ and the stiffness ratio $E_p/E_S$ increase, the normalized pile head stiffness coefficient $F_v$ increases and that for Poisson’s ratio $\nu_S = 0.0$ is larger than that for $\nu_S = 0.5$. It is assumed conventionally that a square pile is idealized as a circular pile with the same cross-sectional area, for example, Small and Zhang [17]. It is found from Figures 9 and 11 that the difference of settlement between the square and circular piles for vertical loads is insignificant. There is an appreciable difference between the results obtained from Seo et al. [39] and those presented by the present method.

Figure 12 shows relationships between the length–diameter ratio $L/d_0$ and the normalized pile head stiffness coefficient $F_v$ for a pile with a rectangular cross section of the aspect ratio $B_x/B_y = 5$. As the length–diameter ratio $L/d_0$ and the stiffness ratio $E_p/E_S$ increase, the normalized pile head stiffness coefficient increases and that for Poisson’s ratio $\nu_S = 0.0$ is larger than that for $\nu_S = 0.5$. Compared
with the results of the pile with a square cross section shown in Figure 11, the normalized pile head stiffness coefficient of the pile with a rectangular cross section of the aspect ratio $B_x/B_y = 5$ is larger than that of the pile with a square cross section. There is a comparative difference between the results obtained from Seo et al. [39] and those given by the present method.

Figures 13 and 14 show relationships between the aspect ratio $B_x/B_y$ and the normalized pile head stiffness coefficient $F_v$ for a pile with a rectangular cross section in the case of Poisson’s ratios $\nu_S = 0.0$ and $\nu_S = 0.5$, respectively. As the aspect ratio $B_x/B_y$ increases, the normalized pile head stiffness coefficient $F_v$ increases.

Figure 13. Relationships between $B_x/B_y$ and the normalized pile head stiffness coefficient $F_v$ for the rectangular pile.

Figure 14. Relationships between $B_x/B_y$ and the normalized pile head stiffness coefficient $F_v$ for the rectangular pile.
stiffness coefficient $F_v$ increases with the increases of the stiffness ratio $E_p/E_S$ and the length–diameter ratio $L/d_e$. The normalized pile head stiffness coefficient of the pile with a rectangular cross section for Poisson’s ratio $v_S = 0.0$ is larger than that of the pile with a rectangular cross section for $v_S = 0.5$ at each aspect ratio $B_x/B_y$. There is a comparative difference between the results obtained from Seo et al. [39] and those presented by the present method.

Therefore, because it is found from Figures 3–6 that the vertical stiffness coefficient of a square pile is slightly larger than that of a circular pile and that of a rectangular pile increases as the aspect ratio of the rectangular pile cross section increases, the difference of the normalized pile head stiffness coefficient between the square and circular piles is insignificant, and that of the rectangular pile increases as the aspect ratio of the rectangular pile cross section increases, as shown in Figures 9–14.

Figure 15 shows relationships between the length–diameter ratio $L/d_e$ and the normalized pile head stiffness coefficient $F_v$ for rectangular, square, and circular piles for Poisson’s ratio $v_S = 0.5$. As the length–diameter ratio $L/d_e$ and the stiffness ratio $E_p/E_S$ increase, the normalized pile head stiffness coefficients $F_v$ increase. For circular piles, the results obtained from the present method are almost same as those given by Mattes and Poulous [28]. Comparing the results between rectangular, square, and circular piles obtained from the present method, the normalized pile head stiffness coefficient of the square pile is slightly larger than that of the circular pile, and the normalized pile head stiffness coefficient of the rectangular pile with the aspect ratio $B_x/B_y = 5$ is larger than that of the square pile. The result obtained from Seo et al. [39] shows that the normalized pile head stiffness coefficient of a square pile is larger than that of a circular pile, and the increase of the normalized pile head stiffness coefficient with the increase of the aspect ratio $B_x/B_y$ for the rectangular pile is negligible in the case of the length–diameter ratios $L/d_e = 25$ and 100.

Figure 16 shows relationships between depth and pile settlement for rectangular and circular piles subjected to the vertical load in a four-layered soil. The elastic properties and thicknesses of the soil layers and Young’s modulus, length, and dimensions of piles used in the present method are same as those adopted by Seo et al. [39]. Compared with the results from the finite element analysis (FEA) given by Seo et al. [39], the displacement from the present method is slightly larger than that.
from FEA for both rectangular and circular piles. The results from the present method are in good agreement with the FEA results.

Figure 17 shows relationships between the horizontal distance from the pile center and the vertical soil displacement of rectangular and circular piles in the soil layers shown in Figure 16. Compared with the results from FEA given by Seo et al. [39], the vertical soil displacements from the present method are similar to those from FEA for both rectangular and circular piles. The results from the present method are in good agreement with the FEA results.

Figure 16. Relationships between the depth and pile settlement for the rectangular and circular piles.

Figure 17. Relationships between horizontal distance and settlement for the rectangular and circular piles.
Let us consider a case history as the first example. Seo et al. [39] presented the computational results using the results of instrumented load tests that Chang and Wong [49] reported. The elastic properties and thicknesses of the soil layers and Young’s modulus, length, and dimensions of piles employed in the present method are same as those adopted by Seo et al. [39], as shown in Figures 18 and 19.

Figure 18 shows measured and predicted relationships between the depth and the axial load for rectangular and circular piles. The axial loads near the pile base predicted from the present method are less than those predicted by Seo et al. [39] for both rectangular and circular piles. The results predicted from the present method are in good agreement with the measured results.

Figure 19 shows predicted and measured relationships between the vertical load and the settlement for rectangular and circular piles. In the article reported by Chang and Wong [49], the reason for the sudden jump in the measured load-settlement curve at 3 MN is not mentioned. The settlement predicted from the present method is slightly larger than that from Seo et al. [39] for both rectangular and circular piles. The results from the present method are in good agreement with the measured data.

Figure 18. Relationships between depth and axial load for the pile load test.

Figure 19. Relationships between vertical load and settlement for the rectangular and circular piles.
Next, we consider a rectangular pile as the second example. Figure 20 shows the geotechnical profile for the pile load test presented in the literature [50]. The top 13.5 m of the soil profile is clay [average $N_{spt} = 1$, where $N_{spt}$ is the standard penetration test blow count]; underneath this layer, there are the sand layer [average $N_{spt} = 15$] with 7.1-m thickness, the silt layer [average $N_{spt} = 35$] with 3.1-m thickness, and the sand layer [average $N_{spt} = 50$] with 9.3-m thickness. In the following presentation, the second sand layer is assumed to be 32.3-m thick for the analysis. The rectangular pile is 28 m in length and the dimensions $B_x = 1.80$ m and $B_y = 0.6$ m in $x$ and $y$ directions, respectively. The Young’s modulus of the pile is 21.5 GPa. It is assumed that the Young’s modulus of the soil layer is $E_s = 3N_{spt}$ (MPa) using the equation presented by Poulos [51]. The Poisson’s ratio is assumed to be 0.3 for the sand and silt layers and 0.5 for the clay layer.

Figure 21 shows predicted and measured relationships between the vertical load and the pile head settlement for the rectangular pile. The settlement predicted from the present method is slightly larger than that measured in the initial stages of loading. The results from the present method are in fairly good agreement with the measured data in the initial stages of loading.
Figure 22 shows measured and predicted relationships between the depth and the axial load for the rectangular pile. Although the axial loads near the middepth of the pile predicted are relatively larger than those measured, the axial loads near the pile base predicted are almost the same as those measured for both axial loads 5 and 10 MN at the pile head. The results predicted from the present method are in good agreement overall with the measured results.

4. CONCLUSIONS

The following conclusions can be drawn from the present investigation:

1) For vertically loaded piles with a rectangular cross section, the settlement influence factor of normal piles (with the length–diameter ratio of 10 or more) in nonhomogeneous soils is derived from Mindlin’s solution for elastic continuum analysis. For short piles with rectangular and circular cross sections subjected to vertical loads, the modified forms of the settlement influence factor of normal piles are produced taking into account the load transfer parameter proposed by Randolph for short circular piles.

2) The modulus of subgrade reaction along the rectangular pile in nonhomogeneous soils for vertical loads is expressed by the settlement influence factor related to Mindlin’s solution in elastic continuum analysis to combine the elastic continuum approach with the subgrade-reaction approach.

3) The relationship between settlement and vertical load for a rectangular pile subjected to a vertical load in nonhomogeneous soils is obtained using the recurrence equation for each layer.

4) The formulation of settlement of soils surrounding a rectangular pile subjected to vertical loads in nonhomogeneous soils is proposed by taking into account Mindlin’s solution for a homogeneous soil and both the equivalent thickness and the equivalent elastic modulus for layers in the equivalent elastic method.

5) By using the first four conclusions, a Winkler model approach of rectangular piles is proposed to analyze the settlement of piles with a rectangular cross section subjected to the vertical loads in nonhomogeneous soils.

6) The vertical stiffness coefficient of a pile with a square cross section is slightly larger than that of a pile with a circular cross section, and that of a pile with a rectangular cross section increases as
the aspect ratio that is the ratio between the dimensions in $x$ and $y$ directions increases, in the case of the same cross-sectional area for each pile.

7) An assumption that a square pile is idealized as a circular pile with the same cross-sectional area is investigated, and it is found that the difference of settlement between the square and circular piles for vertical loads is insignificant. The settlement of a pile with a rectangular cross section decreases as the aspect ratio of the rectangular pile cross section increases, in the case of the same cross-sectional area for each pile.

8) The comparison of the results calculated by the present method for rectangular and circular piles in nonhomogeneous soils has shown good agreement with those obtained from the analytical methods and FEM.

REFERENCES


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44. Steinbrenner W. Tafeln zur Setzungsberechnung. Die Straße 1934; 1.


